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THE GEOMETRICAL CONSTRUCTION OF THE REGULAR POLYGONS USING < A RULER AND A COMPASS > & , THE OVERTHROW OF THE THEORY OF THE NON - CONSTRUCTIBILITY

By

Markos Georgallides

¹Larnaca-Cyprus (Expelled from Varosha-Famagusta town occupied By the Barbaric Turks ,
in Aug -1974) Cyprus , Civil - Structural Engineer (NATUA) , Athens .
Email address : [georgallides.marcos\(at\)cytanet.com.cy](mailto:georgallides.marcos(at)cytanet.com.cy)

Abstract : The Method

- a). In the System of a circle $[O, OK]$, of Diameters $KK' \perp aa' = 2R$, $KK' \perp K(E)$, and a Chord $KA = KE_A$ on $K(E)$, Represents The Fixed Twin - Couple of the Triangles KAK' , $KA E_A$. Simultaneously , The Midpoint O_A of side KE_A , The Isosceles triangle KAE_A , The Base Point D_A of Perpendicular AD_A , and The Perpendicular triangles KAD_A , KAK' Are Constantly Related to each other;.....It is Proved
- 1... On Triangle KAK' any Chord KA is related to its Angle φ_A as $KA = 2R \cdot \sin \varphi_A$, and $|KA|^2 = 2R \cdot h_A$, And on the Triangles KAD_A , KAE_A , $\sin \varphi_A = AD_A / KA$ and Difference of $|KA|^2 - |KE_A|^2 = |KA| \cdot |O_{EA}D_A| = \text{Constant}$, where Point O_{EA} is Midpoint of $KE_A = KA$.
- 2... The locus of Any Point B , which Summation of squares $[BK^2 + BE_B^2] = \text{Constant}$ from the Distances to Fixed Points K, E_B , with KB Fixed and equal to k^2 , IS the Circle $[O_{BE}, O_{BE}B]$, at Point O_{BE} , and to Be Distant from the Midpoint O_{EB} of the Side KE_B
 $\rightarrow$ The distance is equal to $\Rightarrow [BK]^2 + [BE_B]^2 = k^2 = 2 \cdot [O_{EB}B]^2 + [KE_B]^2 \leftarrow$
- 3... On Any 2-Even Polygon Set- $[n, n+2]$ - of Sides KA - KB is Determined the Constant Apothems circle & Archimedes 2n-Sided-Vortex Polygon from the $n+1$. Midpoints M_{EAB} , $M_{B.B'}$ of Distances $|E_A - E_B|$, And $|BB'|$ on which Circle $[M_{EAB}, |M_{EAB} \cdot M_{B.B'}|]$ at Point M_{EAB} and Radius $|M_{EAB} \cdot M_{B.B'}|$ Cuts $[O, OK]$ Circle At Point C such that Side KC Is the In-Between ODD-Polygon. [Fig-6-4-].

The Prior :

a...The Unsolvability of the Three famous Ancient Greek Problems—Trisecting the Angle Doubling the Cube , Squaring the Circle and The Construction of the Regular Polygons **With a Ruler and a Compass** , **Stands on the Base justification of the Algebraic Field Theory (specifically Galois Theory)** and the **Theory of Constructible Numbers** , which were developed in the 19 th century .

b... The Greeks often used other Techniques (like **Conic sections** or **Mechanical tools**) to Solve these Problems , But **their self-imposed "Euclidean" constraint with a Ruler and a Compass** ,(straightedge and compass) made these Specific Problems impossible to solve.

c...In the Published Articles [123],[124],[126] , it is **clearly evident** that the Solution of the Ancient , Unsolved Greek Problems, using **a Ruler and a Compass** [as the constraint has been set by Euclid] , **Has Become Possible**, and is in the Critique of Both , **Human Logic Thinking and , The Artificial Intelligence** when it uses **The Path of Knowledge** to the Truths of Nature , **and which is the Dialectic Logic** of the Euclidean Geometry .

In article [123] , are given on the **2-Vectors , 3-Poles Rotation Squares Mechanism** , where $\vec{E} = \vec{M}$, such the Geometrical as the Mechanical Proof , By using the **Conjugate circles of Polhode and Herpolhode** which consist the Spin of Photons and which is their motion .The frequency needed for the velocity vector $\vec{c}$ to **Rotate** , is used from Kepler's **Unit meter of Time** $k = f^2_e a^3$, where , $a = \lambda / 2$,and which is **the clock** measuring the changes of motions,

In article [124] , It was Noted that **When** in Photons , $\vec{E} = \vec{M}$, **these acquire the common Plane-meter** the Square $CMNH = [CM]^2 = \pi.[EC]^2$, where EC is the circle's Radius , **Using the Bellow-Motion Method** by creating the Square $CMNH = [CM]^2$ from E , M , and from $[E/\sqrt{2}]^2 = \pi.[M/\sqrt{2}]^2$, $\pi \equiv \frac{[E/\sqrt{2}]^2}{[M/\sqrt{2}]^2}$ **and Thus Photon is Squared to an UNID SQUARE .**

In article [126] , It is important to Note that **When** in Photons , $E \neq M$, **these acquire the common Space-meter** $^3\sqrt{2}$, **Using Quantum-Cloning Method** for creating a Perfect-Copy of E , M , as are when $E = 2.M$ and , $[E \neq M]^3 = 2.[\vec{E} = 2.\vec{M}]^3$. **With this Way Photon is Duplicated to a New-One** . The simplified But a Rigorous Proof of the Problem , *The Squaring of the Circle using a Ruler and a Compass* , as it was first Posed by the Ancient Greeks , is followed by The Photons which Square their , Energy circle $\equiv$ The **Herpolhode** $\equiv$ **SPIN** , to equal an **Energy Unit Square** , which **UNIT-Square** they Promote , **common Plane - meter** , either as the *Speed of the Photon which is their Electric Field* , or they store it Perpendicularly to the motion in an equal area , *The Anti-Square which is their Magnetic Field* . Promotion is done at the **Birefringence angle** of 45° , and thus The **Bellow-motion** is their Torsional motion . At Phase Angle where $\vec{E} = 2.\vec{M}$,Energy Cube $[M/2]^3$ is Duplicated and enters INTO . $[E]^3 = [2.M]^3 = 8.M^3 \equiv$ **Tetrahedron-Cube-Sphere-Mechanism**. [102]

d...From [40]--The Special Problems of Euclidean Geometry [47] consist the , *Moulds of Quantization* , of E - Geometry in it , to become $\rightarrow$ **Monad** , through mould of Space Anti-Space in itself , *which is the material Dipole in inner monad Structure and which is identical with the Electromagnetic cycloidal field* $\rightarrow$ **Linearly** through the mould of the **Parallel Theorem** [44- 45] , *which are the equal distances between Points of Parallel and line* $\rightarrow$ **In Plane** , through mould of **Squaring the circle** [46] , *where the Two Equal and Perpendicular Monad-Vectors , $E = M$, consist a Plane acquiring The common Plane -meter, π , and in Space (volume) through mould of Duplication of the Cube [46] , where*

Any Two Unequal Perpendicular monads $E \neq M$, acquire the common Space-meter $^3\sqrt{2}$, Using Quantum-Cloning Method to be Twice each other as analytically Proved-explained. The Unification of $\rightarrow$ Space and Energy $\leftarrow$ Becomes through [STPL] Geometrical Mould Mechanism of Elements, the minimum Energy - Quanta, In monads $\rightarrow$ Particles, Anti - Particles, Bosons, Gravity - Force, Gravity - Field, Photons, Dark Matter, and Dark - Energy, consisting the Material Dipoles in inner monad Structures, i.e.

$\rightarrow$ the innate Electromagnetic Cycloidal Field of monads $\leftarrow$ [39-41]

Euclid's Elements consist of assuming a small Set of intuitively appealing Axioms, Proving many other Propositions. Because No One until [9] succeeded to Prove the Parallel Postulate By means of **Pure Geometric Logic**, many self consistent Non -Euclidean Geometries have been discovered, Based on Definitions, Axioms or Postulates, in order that Non of them contradicts any of the other Postulates. It was Proved [39], that the only Space - Energy Geometry is Euclidean, agreeing with the **Physical Reality** on Unit $AB \equiv$ Segment $\equiv$ Vector which is The Electromagnetic field of the Quantized on $\overline{AB}$ Energy Space Vector of Angular Momentum = Spin, on the contrary to the General Relativity of Space-time which is based on the Rays of the Non-Euclidean Geometries to the limited velocity of light in Planck's cavity. Euclidean Geometry elucidated the Definitions of its geometry - content, i.e. { [for Point, Segment, Straight Line, Plane, Volume, Space [S], Anti-space [AS], Sub - space [SS], Cave, The Space-Anti-Space Mechanism of the Six-Triple-Points-Line, that Produces and transfers Points of Spaces, Anti-Spaces and Sub-Spaces in a Common Inertial Sub-Space, and a cylinder, in Gravity field [MFMF] Particles} and describes the Space-Energy vacuum beyond Plank's length level [Gravity's Length $3,969.10^{-62}$ m], reaching the absolute Point

$L_v = e^{i(\frac{N\pi}{2})b=10^{-N}} = -\infty$ m = 0 m, which is Nothing, and the Absolute

Primary Neutral Space [PNS] = cave [$r = 10^{-35} \sim 10^{-62}$ m [43-46].

In Physics, there is No single Process called "Duplication of Energy" for Photons Because Energy must always be conserved, or when Two Photons are "merged" into a single Photon.

In Mechanics, The Gravity-cave Energy Volume quantity $|\bar{c}| \equiv [wr]$ is Doubled, and is Quantized in Planck's - cave Space quantity $(h/2\pi) =$ The Spin $= 2.[wr]^3 \rightarrow$ i.e. Energy Space quantity $[wr]$ is Quantized, doubled, and becomes the Space quantity h/π following Euclidean Space-moulds of Duplication of the cube, in Sphere volume $V = (4\pi/3).[wr]^3$ and follows the Squaring of the circle π , and in Sub-Space-Sphere volume $^3\sqrt{2}$, as Trisection.

Keywords : The Quadrature of The Electromagnetic Fields in Dual - Photons,

- $\rightarrow$ The Squaring of the circle, Using a Ruler and a Compass [124]
- The Dublication of Photon, Using The Quantum - Cloning Method,
- $\rightarrow$ The Dublication of the Cube, Using A Ruler and a Compass [126]
- The Trisection of the Energy Octave, Using an Directional -Triangle
- $\rightarrow$ The Trisection of any Angle, Using a Ruler and a Compass [128]
- The Construction of The Regular-Polygons Using 2-Directional-Triangles
- $\rightarrow$ The Construction of Regular-Polygons Using a Ruler & a Compass [130]
- THE NEEDED Directional -Triangles FOR,
- $\rightarrow$ (1) The Squaring of the circle, Using a Ruler and a Compass [124]
- $\rightarrow$ (2) The Dublication of the Cube, Using a Ruler and a Compass [126]
- $\rightarrow$ (3) The Trisection of any Angle, Using a Ruler and a Compass [128]
- $\rightarrow$ (4) The Construction of Regular-Polygons Using Ruler & Compass [130]

-
- 1... The **Isosceles Right Angle Directional -Triangle** which **Hypotenuse** lies on the Diameter of the circle with relation of Base -Angles $1 : 2 = 45^\circ / 90^\circ$, & it is the **Material-Mould for the Squaring The circle & Energy-circle** [124].
 - 2.. The **Right Angle Directional - Triangle** which **Hypotenuse** lies on the Diameter of the circle with relation of Base - Angles $1 : 2 = 30^\circ / 60^\circ$, and it is [126].
→ **The Material - Mould for The Dublication of the Energy Cube** ←
 - 3.. The **Right Angle Directional - Triangle** which **Hypotenuse** lies on the Diameter of the circle with relation of Base - Angles $1 : 2 = 30^\circ / 60^\circ$, and it is the **Material-Mould forThe Trisection of any Angle** ← & → **The Trisection of Energy - Octava** ← [128] .
 - 4... The **Isosceles Directional - Triangle** which **Base is a Chord** of the circle with relation of Base - Angles $\varphi_A + 2.\varphi_{EA} = 180^\circ$, and it is the , **Material-Mould for Constructing the Regular Polygons of the circle .** & The Polygons of the **Energy-circle on STPL Forces .**
The **Two Right Angle** at Points **B , B'** **Directional - Triangles** which **Foot** form Their **Apothems** on Archimedes 2-n.Sided Vertices Circle to **B , B'** arc . [130].

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A... THE ALGEBRAIC SOLUTION : [63]

It has been Proved by De Moivre's , that the n-th roots on the unit circle AB are represented by the vertices of the Regular n-sided Polygon inscribed in the circle .

It has been Proved that the Resemblance Ratio of Areas , of the circumscribed to the inscribed squares (Regular quadrilateral) which is equal to 2 , leads to the squaring of the circle. [124]

It has been also proved that , Projecting the vertices of the Regular n-Polygon on any tangent of the circle , then the Sum of the Heights y_n is equal to $n * R$. [63M]

This is a linear relation between Heights , h , and the radius R , of the circle , the monad .

This property on the circle yields to the Geometrical construction (As Resemblance Ratio of Areas is now controlled) , and the Algebraic measuring of the Regular Polygons as follows :

For : R = The Radius of the circle , with a random Diameter AB .

a = The Side of the Regular n -Polygon inscribed in the circle

n = Number of sides , a , of the n -Polygon , then exists :

$$n \cdot R = 2 \cdot R + 2 \cdot y_1 + 2 \cdot y_2 + 2 \cdot y_3 + \dots + 2 \cdot y_n \dots (n)$$

The Heights y_n are as follows :

$$y_B = [2 \cdot R]$$

$$y_1 = [4 \cdot R^2 - a^2] / (2 \cdot R)$$

$$y_2 = [4 \cdot R^4 - 4 \cdot R^2 \cdot a^2 + a^4] / (2 \cdot R^3)$$

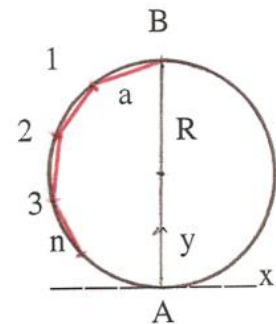


Figure-1-

$$y_3 = \frac{[8 \cdot R^6 - 10 \cdot R^4 \cdot a^2 + 6 \cdot R^2 \cdot a^4 - a^6] - a^2 \cdot \sqrt{64 \cdot R^8 - 96 \cdot R^6 \cdot a^2 + 52 \cdot R^4 \cdot a^4 - 12 \cdot R^2 \cdot a^6 + a^8}}{2 \cdot R^5}$$

$$y_n = [\dots] / 2 \cdot R^n$$

THE ALGEBRAIC EQUATIONS OF THE REGULAR n - POLYGONS

(a) REGULAR TRIANGLE :

The Equation of the vertices of the Regular Triangle is :

$$3 \cdot R = 2 \cdot R + [\frac{4 \cdot R^2 - a^2}{R}] \quad \text{or} \quad >>> \quad R^2 = 4 \cdot R^2 - a^2 \quad >>> \quad a^2 = 3 \cdot R^2$$

$$\text{The side } a_3 = R \cdot \sqrt{3} \dots\dots\dots(1).$$

(b) REGULAR QUADRILATERAL  (SQUARE) :

The Equation of the vertices of the Regular Square gives :

$$4.R = 2.R + \left[\frac{4.R^2 - a^2}{R} \right] \quad \text{or} \quad \gg \gg \quad a^2 = 2 \cdot R^2$$

$$\text{The side } a_4 = R \cdot \sqrt{2} \dots\dots\dots(2)$$

(c) REGULAR PENTAGON  :

The Equation of the vertices of the Regular Pentagon is :

$$5.R = 2.R + \left[\frac{4.R^2 - a^2}{R} \right] + \left[\frac{4.R^4 - 4.R^2 \cdot a^2 + a^4}{R^3} \right] \gg \gg a^4 - 5.R^2 \cdot a^2 + 5.R^4 = 0$$

Solving the equation gives :

Solving the equation gives :

$$R^2 = \frac{5.R^2 - \sqrt{25.R^4 - 20.R^4}}{2} = \frac{5.R^2 - R^2 \cdot \sqrt{5}}{2} = \left[\frac{5.R^2 - R^2 \cdot \sqrt{5}}{2} \right] = \frac{R^2}{2} (5 - \sqrt{5}) \dots\dots(3)$$

$$a^2 = \{ R^2 \} \cdot [10 - 2\sqrt{5}] \gg \gg \quad \text{The side } a_5 = \left| \frac{R}{2} \right| \sqrt{10 - 2\sqrt{5}} \dots\dots(d)$$

REGULAR HEXAGON  :

The Equation of the vertices of the Regular Hexagon is :

$$6.R = 2.R + \left[\frac{4.R^2 - a^2}{R} \right] + \left[\frac{4.R^4 - 4.R^2 \cdot a^2 + a^4}{R^3} \right] \gg a^4 - 5.R^2 \cdot a^2 + 4.R^4 = 0$$

Solving the equation gives :

$$a^2 = \frac{5.R^2 - \sqrt{25.R^4 - 16.R^4}}{2} = \left[\frac{5 - 3}{2} \right] \cdot R^2 = R^2 \quad \text{The side } a_6 = R \dots\dots(4)$$

(e) REGULAR HEPTAGON  :

The Equation of the vertices of the Regular Heptagon is :

$$7. R = 2. R + \left[\frac{4. R^2 - a^2}{R} \right] + \left[\frac{4. R^4 - 4. R^2 . a^2 + a^4}{R^3} \right] + \left[\frac{8. R^6 - 10. R^4 . a^2 + 6. R^2 . a^4 - a^6}{2. R^5} \right] -$$

$$- \left[\frac{a^2}{2. R^5} \right] . \sqrt{64 . R^8 - 96. R^6 . a^2 + 52 . R^4 . a^4 - 12. R^2 . a^6 + a^8}$$

Rearranging the terms and solving the equation in the quantity **a** , obtaining :

$$R^2 . a^{10} - 13 . R^4 . a^8 + 63 . R^6 . a^6 - 140 . R^8 . R^4 + 140 , R^{10} . a^2 - 49 . R^{12} = 0 , \text{ for } a^2 = x$$

$$x^5 - 13 . R^2 . x^4 + 63 . R^4 . x^3 - 140 . R^6 . x^2 + 140 . R^8 . x^1 - 49 . R^{10} = 0 \quad \dots(7)$$

Solving the 5 nth degree equation the Real roots are the following two :

$$x_1 = R^2 . [3 - \sqrt{2}] , \quad x_2 = R^2 . [3 + \sqrt{2}] \quad \text{which satisfy equation (7)}$$

Having the two roots , the Sum of roots be equal to 13 , their combination taken 2,3,4 at time be equal to 63 , - 140 , 140 , the product of roots be equal to - 49 , then equation (7) is reduced to the third degree equation as :

$$z^3 - 7. z^2 + 14. z - 7 = 0 \quad \dots(7a)$$

by setting $\psi = z - (-7/3)$ into (7a) , then gives $\psi^3 + \rho . \psi + q = 0 \quad \dots (7b)$ where ,

$$\rho = 14 - (-7)^2 / 3 = 14 - 49/3 = - 7 / 3 \quad > \quad \rho^2 = 49 / 9 \quad > \quad \rho^3 = - 343 / 27$$

$$q = 2. (-7)^3 / 27 + 14 . (-7) / 3 - 7 = 7 / 27 \quad > \quad q^2 = 49 / 729$$

Substituting ρ , q then $\psi^3 - (7/3) . \psi + (7/27) = 0 \dots (7b)$

The solution of this third degree equation (7b) is as follows : $\rho = - 7 / 3$
 $q = 7 / 27$

$$\text{Discriminant } D = q^2 / 4 + \rho^3 / 27 = (49 / 729 . 4) - (343 / 27.27) = - [49 / 108] < 0$$

$$D = - 49 / 108 = i^2 (3.21^2 / 4 . 27^2) = i^2 (21 . \sqrt{3} / 2 . 27)^2 = i^2 (21. \sqrt{3} / 54)^2$$

$$D = [7 . \sqrt{3} / 18]^2 . i^2 \quad \text{and also} \quad \sqrt[2]{D} = \frac{| 7 . \sqrt{3} |}{18} . i$$

Therefore the equation has three Real Roots :

$$\text{Substituting } \psi = w - \rho / 3 . w = w + 7 / 9 . w \quad > \quad \psi^2 = w^2 + 49 / 81 . w^2 + 14 / 9$$

$\psi^3 = w^3 + 343/729w^3 + 49/27w + 7w/3$

to (7b) then becomes $w^3 + 343/729 w^3 + 7/27 = 0$
 and for $z = w^3$ $z + 343/729 z + 7/27 = 0$

$$z^2 + 7.z/27 + 343/729 = 0 \quad \dots(7c)$$

The Determinant $D < 0$ therefore the two quadratic complex roots are as follows :

$$\begin{aligned} Z1 &= \left[-7/27 - \sqrt{49/27.27 - 4.343/729} \right] / 2 = \left[-7/27 - \sqrt{49/27.27.4 - 49.7.4/27.27.4} \right] / 2 \\ &= \left[-7/27 - \sqrt{(49 - 49.28)/27.27.4} \right] / 2 = \left[-7 - 7.\sqrt{-27} \right] / 27.2 \\ &= \left[-7 - 21.\sqrt{-3} \right] / 3^3.2 = \left[-7 \right] / 2 \cdot (1 - 3.i.\sqrt{3}) / 27 = (-7/54).[1 - 3.i.\sqrt{3}] \\ Z2 &= \left[-7/2.(1 - 3.i.\sqrt{3}) \right] / 27 = (-7/54).[1 + 3.i.\sqrt{3}] \end{aligned}$$

The Process is beginning from the last denoting quantities to the first ones :

$$\text{Root } W_{1,2} = \sqrt[3]{-7 \pm 21.i.\sqrt{3}} = \sqrt[3]{-7 \pm 21.i.\sqrt{3}} = \sqrt[3]{-7 \pm 21.i.\sqrt{3}} \dots(1)$$

$$\text{Root } \psi = W + 7/9.W = \sqrt[3]{-7 \pm 21.i.\sqrt{3}} + \sqrt[3]{-7 \pm 21.i.\sqrt{3}} \dots(2)$$

$$\text{Root } X = \psi - \rho/3 = \psi + 7/3 = \sqrt[3]{-7 \pm 21.i.\sqrt{3}} + \sqrt[3]{-7 \pm 21.i.\sqrt{3}} \dots(3)$$

$$X = \frac{1}{3} \left[\sqrt[3]{-7 \pm 21.i.\sqrt{3}} + \sqrt[3]{-7 \pm 21.i.\sqrt{3}} + 7 \right] \cdot R^2$$

The root a_7 of equation (7) equal to the side of the Regular Heptagon is $a_7 = \sqrt{X}$

$$\frac{2}{\sqrt{X}} = \frac{1}{\sqrt{X}} \left[\sqrt[3]{-7 \pm 21.i.\sqrt{3}} + \sqrt[3]{-7 \pm 21.i.\sqrt{3}} + 7 \right] \cdot R^2$$

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The Geometrical Construction of All - Even & Odd Regular Polygons .

$$a_7 = \frac{1}{\sqrt[3]{\frac{7}{2} \sqrt{\frac{-7 \pm 21i\sqrt{3}}{2}} + \frac{-7 \pm 21i\sqrt{3}}{2} + 7}} \quad \text{R} \dots (4)$$

Instead of substituting $\psi = w - \rho/3.w$ into (7.b), is substituted $\psi = u + v$ and then gives the equation of second degree as $z^2 + 7z/27 + 343/729 = 0$ which has the two complex roots as follows :

$$z_{1,2} = \frac{7}{54} \cdot [-1 \pm 3i\sqrt{3}] = \frac{1}{27} \cdot [(-7 \pm 21i\sqrt{3})/2] \text{ and the side } a_7 \text{ is as ,}$$

$$a_7 = \frac{2}{\sqrt[3]{z_1} + \sqrt[3]{z_2} + \frac{7}{3}} \quad \text{and by substituting } Z_1, Z_2 \text{ into (7b) becomes the same formula as in (4) .}$$

$$\text{It is easy to see that } \frac{1}{\sqrt[3]{-(7/2) \cdot [1 - 3i\sqrt{3}]}} * \frac{1}{\sqrt[3]{-(7/2) \cdot [1 + 3i\sqrt{3}]}} = 7$$

Analytically is :

$$x = \frac{1}{R^2} \cdot [0,753\,020\,375\,967\,025\,701\,777] \gg x^2 = 0,56704$$

$$a_7 = \sqrt[3]{x} = R \cdot [0,867\,767\,453\,193\,664\,601 \dots]$$

By using the formula of the Real Root of equation (7a) then :

$$a.x^3 + b.x^2 + c.x + d = 0 \gg \text{for } a = 1, b = -7, c = 14, d = -7 \text{ then } x^3 - 7x^2 + 14x - 7 = 0$$

$$x = -\frac{b}{3} \frac{2 \sqrt[3]{-b^2 + 3c}}{[3(-2b^3 + 9bc - 27d + \sqrt{4(-b^2 + 3c)^3 + (-2b^3 + 9bc - 27d)^2}]^{1/3}} + \frac{[-2b^3 + 9bc - 27d + \sqrt{4(-b^2 + 3c)^3 + (-2b^3 + 9bc - 27d)^2}]}{32 \sqrt[3]{3}} .$$

Substituting the coefficients to the upper equation becomes :

$$-b^2 + 3c = -(-7)^2 + 3 \cdot 14 = -49 + 42 = -7$$

$$-2b^3 + 9bc - 27d = -2 \cdot (-7)^3 + 9 \cdot (-7) \cdot 14 - 27 \cdot (-7) = 686 - 882 + 189 = -7$$

$$4.(-b^2+3.c)^3 = 4 (-7)^3 = -1372 (-2.b^3+9.b.c-27.d)^2 = (-7)^2 = 4932^{1/3} = \sqrt[3]{8.4} = 2.\sqrt[3]{4}$$

$$X = \frac{7}{3} - \frac{\sqrt[3]{2} \cdot (-7)}{3 \cdot \sqrt[3]{-7+21.i.\sqrt{3}}} + \frac{\sqrt[3]{-7+21.i.\sqrt{3}}}{2 \cdot \sqrt[3]{4}} \quad \text{and}$$

$$a_7 = \sqrt[3]{X} = \sqrt[3]{\frac{7}{3} + \frac{7 \cdot \sqrt[3]{2}}{3 \cdot \sqrt[3]{-7+21.i.\sqrt{3}}} + \frac{\sqrt[3]{-7+21.i.\sqrt{3}}}{2 \cdot \sqrt[3]{4}}} \quad \begin{array}{l} \text{The Side of the} \\ \text{Regular Heptagon} \\ \text{(4.a)} \\ \text{Further Analysis to the Reader} \end{array}$$

(f) REGULAR OCTAGON  :

The equation of vertices of the Regular Octagon is

$$8.R = 2.R + (a^2) + \frac{(4.R^2 \cdot a^2 - a^4)}{R^3} + \frac{10.R^4 \cdot a^2 - 6.R^2 \cdot a^4 + a^6 + a^2 \cdot \sqrt{64.R^8 - 96.R^6 a^2 + 52.R^4 \cdot a^4 - 12.R^2 \cdot a^6 + a^8}}{2 \cdot R^5}$$

Rearranging the terms and solving the equation in the quantity **a**, is a 10th degree equation, and by reduction ($x = a^2$) is find the 5th degree equation as follows :

The equation of vertices of the Regular Octagon is

$$a^{10} - 13.R^2 \cdot a^8 + 62.R^4 \cdot a^6 - 132.R^6 \cdot a^4 + 120.R^8 \cdot a^2 - 36.R^{10} = 0$$

$$x^5 - 13.R^2 \cdot x^4 + 62.R^4 \cdot x^3 - 132.R^6 \cdot x^2 + 120.R^8 \cdot x - 36.R^{10} = 0 \dots (a)$$

Solving the 5th degree equation is find the known algebraic root of Octagon of side **a** as

The roots are >>>>>>>>>> $x_1 = R^2 \cdot [2 - \sqrt{2}]$, $x_2 = R^2 \cdot [3 - \sqrt{3}]$

$$a_8 = \sqrt{x} = R \cdot \sqrt{2 - \sqrt{2}} \dots (b)$$

Verification :

$$x = a^2 = R^2 (2 - \sqrt{2}) \quad x^2 = R^4 \cdot (6 - 4\sqrt{2}) \quad x^3 = R^6 \cdot (20 - 14\sqrt{2})$$

$$x^4 = R^8 \cdot (68 - 48\sqrt{2}) \quad x^5 = R^{10} \cdot (232 - 164\sqrt{2}) \quad \dots\dots\dots(c)$$

by substitution (c) in (a) becomes :

$$R^{10} \cdot [232 - 164 \cdot \sqrt{2}] = R^{10} \cdot [232 - 164 \cdot \sqrt{2}^2]$$

$$- R^{10} \cdot [884 - 624 \cdot \sqrt{2}] = R^{10} \cdot [-884 + 624 \cdot \sqrt{2}]$$

$$R^{10} \cdot [1240 - 868 \cdot \sqrt{2}] = R^{10} \cdot [1240 - 868 \cdot \sqrt{2}]$$

$$- R^{10} \cdot [792 - 528 \cdot \sqrt{2}] = R^{10} \cdot [-792 + 528 \cdot \sqrt{2}]$$

$$R^{10} \cdot [240 - 120 \cdot \sqrt{2}] = R^{10} \cdot [240 - 120 \cdot \sqrt{2}]$$

$$- R^{10} \cdot [36 \quad \quad \quad] = R^{10} \cdot [-36 \quad \quad \quad]$$

$$R^{10} \cdot [1712 - 1712 + (1152 - 1152) \cdot \sqrt{2}] = 0$$

$$R^{10} \cdot [0+0] = 0 \quad \text{therefore Side } a_8 = R \cdot \sqrt{2} - \sqrt{2} \quad \dots\dots (b)$$

CONCLUSIONS :

By summation the heights y on any tangent in a circle ,which hold for every Regular n-sided Polygon inscribed in the circle as the next is :

$$n \cdot R = 2 \cdot R + 2 \cdot y_1 + 2 \cdot y_2 + 2 \cdot y_3 + \dots\dots 2 \cdot y_n \quad \dots\dots\dots(n)$$

the sides a_n of all these Regular n-sided Polygons are Algebraically expressed .

The Geometrical Construction of all Regular Polygons has been proved to be based on the solution of the moving Segment ZD of the figure of page 8 and it is the Master Key of Geometry , because so , the nth degree equations are the vertices of the n-polygon .

In this way , all Regular p - gon are constructible and measureable .

The mathematical reasoning is based on Geometrical logic exclusively alone .

As the Resemblance Ratio of Areas on the 4 - gone is equal to 2 , the Problem of squaring

the circle has been approached and solved [124] by extending Euclid logic of Units (under the restrictions imposed to seek the solution , with a ruler and a compass ,) on the unit circle

AB , to unknown and now the Geometrical elements . (the settled age-old question for all these problems is not valid) .

The Regular Heptagon :

According to Heron , the Regular Heptagon is equal to six times the equilateral triangle with the same side and is the approximate value of $\sqrt{3} \cdot R / 2$

According to Archimedes , given a straight line AB we mark upon it two points C , D such that $AD \cdot CD = DB^2$ and $CB \cdot DB = AC^2$, without giving the way of marking the two Points .

According to the Contemporary Method , the side of the Regular Heptagon is the Root of a third degree equation with three real roots , one of which is that of the Regular Heptagon as analytically Presented.

B.1... THE GEOMETRICAL SOLUTION OF THE POLYGONS :

b-1.. The Even and Odd n-Polygons Properties : [63]

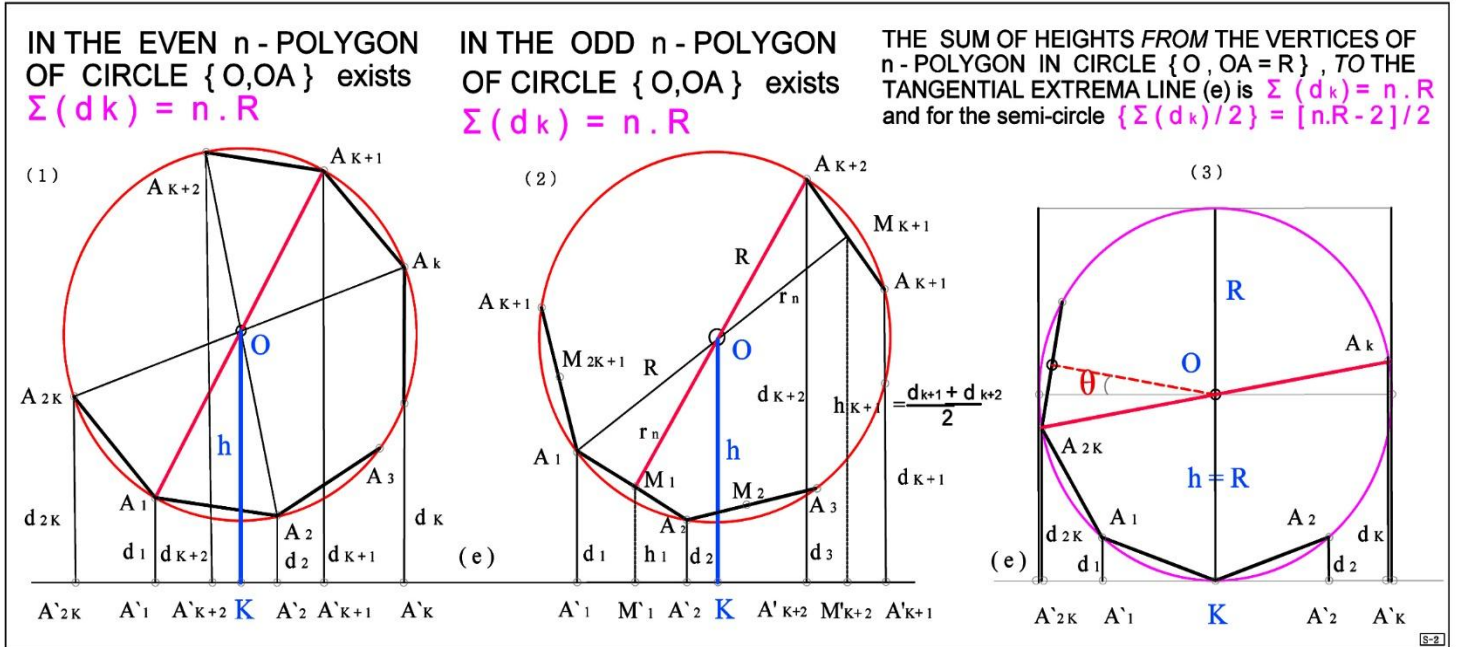


Figure-2- An Even and an Odd n -Polygon in circle $[O, OA]$ with diameters , $A_k A_{2k}$, Passing from A_{2k} , as Vertex (apex) of the Polygone and Diameters , $A_{k+2} M_1$, Perpendicular to Side $A_1 A_2$.

Let Be the n -Polygon $A_1, A_2, A_3, A_k, A_{k+1}, A_{k+2}, A_{2k}$, in circle (O, OA_1) ,
(e) a straight line not intersecting the circle ,

d_1, d_2, d_{2k} , The Heights of the Vertices to (e) line ,

h_1, h_2, h_{2k+1} , The Heights of the midpoints $M_k M_{k+1}$ of the Sides to (e) line and

$OK = h$, The Height from the center O to (e) line .

$n_1, n_3, n_5, \dots, n_{n+1}$, The Number of Sides in an Odd Regular – Polygon.

$n_2, n_4, n_6, \dots, n_n$, The Number of Sides in an Even Regular – Polygon.

To Proof :

In any n - Polygon , The Sum , $\Sigma = \Sigma (h)$, of the Heights , d_1, d_2, d_{2k} , of the Vertices $A_1, A_2, A_3, A_k, A_{k+1}, A_{k+2}, A_{2k}$, where $n = 2k$, from any straight line (e) is equal to

$$\Sigma = \Sigma (h) = n \cdot OK = n \cdot h$$

The Proof Fig-1- :

From any Vertex A_k , of the n-Polygon Draw the Diameter ($A_k O A_{2k}$)

a.. When $n = 2.k$ $\rightarrow$ then Vertex A_{2k} belongs to the Polygon

b.. When $n = 2.k + 1$ $\rightarrow$ then line $A_k O$, is mid-perpendicular to one of the sides .

Case a.. $n = 2.k$ Fig-2- (1)

Exists $\frac{n}{2} = \frac{2k}{2} = k$, and are the Pairs of vertices in opposite diameters as A_1, A_{k+1} , and the k , Trapezium which has bases the Heights of the vertices in opposite diameters from (e) line , and which have Height $OK = h$, as Common Height from their Diameter , i.e.

From Trapezium $A_1, A'_1, A_{k+1}, A'_{k+1}$ exists $d_1 + d_{k+1} = 2.h$ and analogically is ,
 $d_2 + d_{k+2} = 2.h$
 $d_3 + d_{k+3} = 2.h \dots d_k + d_{k+1} = 2.h$

And by Summation ,

$$d_1 + d_2 + \dots d_k + d_{2k} = 2.h \text{ or } \Sigma = (2k) \cdot h = n \cdot h = n \cdot OK \dots\dots(1)$$

Case b.. $n = 2.k + 1$ Fig-2--(2)

$A_1 A_2, A_2 A_3, \dots\dots A_{2k+1} A_1$, the Sides of the Polygon .

M_1, M_2, M_{2k+1} , are the Midpoints of sides from line (e)

$h_1, h_2, \dots h_{2k+1}$ The corresponding Heights of midpoints from (e) .

The Diameter from vertex A_1 is Perpendicular to side $A_{k+1} A_{k+2}$ which has the midpoint M_{k+1} , while the other $A_1 M_{k+1} = A_1 O + OM_{k+1} = R + r_n$, The Radius plus the The Apothem In Trapezium $A_1 A'_1 M'_{k+1} M_{k+1}$ with Bases $A_1 A'_1$ and $M'_{k+1} M_{k+1}$, both Perpendicular to (e) line is Parallel to Height $OK = h$ and bisects $A_1 O = R$ and $OM_{k+1} = r_n$ and from figure , exists

$$OK = h = \frac{R h_{k+1} + r_n \cdot d_1}{R + r_n} \dots\dots\dots(2) \quad \text{i.e.}$$

Height OK is common to all $2k+1$ Trapezium which are formed as $A_1 A'_1 M'_{k+1} M_{k+1}$ and

OK Height divides also the corresponding to $A_1 M_{k+1}$ side with the same analogy as $\frac{R}{r_n}$.

By Summation of $2k+1$ Parts of (2) which are all equal to $OK = h$, then from the $2k+1$ different Between them Trapezium referred exists ,

The Summation of Heights of the Regular Polygons ,

$$(2k+1). h = \frac{R \{ h_{k+1} + h_{k+2} + h_{k+1+h_1} + \dots h_k \} + r_n \cdot \{ d_1 + d_2 + \dots d_{k+1} + d_{2k+1} \}}{R + r_n} = n \cdot h = \frac{R.S + r_n \Sigma}{R + r_n} \dots\dots(3)$$

Where $S = h_1 + h_2 + \dots h_k + d_{2k+1}$. Since $h_1, h_2, \dots h_k, d_{2k+1}$ are the diameters of Trapezium with bases d_1, d_2 to h_1, d_2, d_3 to h_2 and so on and also d_{2k+1}, d_1 to h_{2k+1} then

$$S = \frac{d_1 + d_2}{2} + \frac{d_2 + d_3}{2} + \dots \frac{d_{2k} + d_1}{2} = \frac{2\{d_1 + d_2 + \dots d_{2k+1}\}}{2} = d_1 + d_2 + \dots d_k + d_{2k} = \Sigma \text{ and}$$

(3) becomes $\rightarrow$ (4)

$$n \cdot h = \frac{R \cdot S + r_n \Sigma}{R + r_n} = \left[\frac{R + r_n}{R + r_n} \right] \cdot \Sigma = \Sigma \text{ i.e. } \Sigma = n \cdot h \text{ for all Even and Odd } n\text{-Polygons}.$$

And is a Relation between Heights and the Number of the Regular Polygons.

Case c.. Line (e) is Extrema as Tangential to circle Fig-2--(3)

In this case height, h , is equal to radius R and $OK = h = R$.

Since the Sum of Heights of the vertices in any n -Polygon is $\Sigma = n \cdot h = n \cdot OK$ then $\Sigma = n \cdot R$

This remark helps to construct Geometrically i.e. with a Ruler and a Compass, all the Regular n -Polygons Because gives the relation of the Mid-Perpendicular to the radius r_n of the inscribed circle which is related to the Interior angle $w = \left\{ \frac{n-2}{n} \right\} \cdot 180^\circ$. $w = \left\{ \frac{360}{n} \right\}$.

i.e. Angles, w , in a circle of radius, R , define the n -Sides, $A_1 A_2$, of the Regular Polygon which in turn define the Sum, Σ , of their Heights equal to $\Sigma = n \cdot R$

Since also the relation of radius R , between the Circle and r , of the Inscribed circle is extended to Heights, this helps Extrema - Method to be applicable on the solution which follows.

The Theory of Means :

It was known from Pappus the how to exhibit in a semicircle all three means, namely, The Arithmetic, The Geometric, and The Harmonic mean.

In Fig.-3--(1a) $\rightarrow$ Is given Segment AC and is defined Harmonic mean **DE** for any BA, BC

Draw at any Point B , **BD** at right angles to AC meeting the circle ($O, OA = OC$)

Join DO and from Point B Draw the Perpendicular **BE** on it.

Show that **DE** is the Harmonic - Mean between **BA, BC**.

The Proof :

Since ABC is a Right - angled triangle, and $DE = AB$ then,

$$AB^2 = AD \cdot AE = AD \cdot (AD + DE) = AD \cdot (AD + AB) = AD^2 + AD \cdot AB \text{ therefore,}$$

$$AD^2 = AB^2 - AD \cdot AB = AB \cdot (AB - AD) \text{ or } AD^2 = AB \cdot MB$$

That is, AM is the Harmonic Mean in AB Segment, or between AB and MB .

In Fig.-3--(1b) $\rightarrow$ On the Diameter AB of circle ($O, OA = OB$), is the Point M .

Draw BC at Right angles to AB meeting circle (B, BO at C).

Join AC meeting circle (C, CB) at Points D, E , and Draw the circle (A, AD) meeting Diameter AB at Point M .

Show that **AM** is the Harmonic - Mean between **AB, BM**

Proof :

Since ADC is a right - angled triangle, and BD is Perpendicular to AB then issues,

$BD^2 = BA \cdot BC$ and since BEO is a right-angled triangle, and BD is Perpendicular to OB

then issues $BD^2 = DE \cdot DO$. By rearranging **DE** $= BD^2 / DO = BD^2 / OA$, because $DO = OA$.

So issues **DE** $= BD^2 / OA = [BA \cdot BC] / OA$ that is,

$\rightarrow$ **DE is the Harmonic Mean Between BA and BC** $\leftarrow$.

b-2.. The Theory , On Segments and Angles Relation :**The Three Circles [O,OK] – [K,KA] – [Δh =Diameter] – Method :**

- In Fig.-3--(2) → Two Even n and $n+2$ Regular Polygons on the same circle (O , OA) where,
- n , $n+2$ are the Number of sides differing by an Even number
 - λ_a = The length of a Side of a – [n - Polygon].
 - λ_b = The length of a Side of b – [$n+2$ Polygon].
 - r_a = Apothem = Απόστημα = Radius of inscribed circle of a –Polygon
 - r_b = Apothem = Απόστημα = Radius of inscribed circle of b –Polygon).
 - h_A = The Height of KA_1 Side of a – Polygon .
 - h_B = The Height of KB_1 Side of b – Polygon .
 - Δh = $h_A - h_B$, the Difference of Heights .
 - Δr = $r_a - r_b$, the Difference of Apothems .
 - S = The Sum of interior angles equal to $(n-2) \cdot 180^\circ =$
 $(n-2) \cdot \pi \frac{h_A}{\lambda_a} = \sin \varphi_a$, $\frac{h_B}{\lambda_b} = \sin \varphi_b$, $\frac{h}{\lambda} = \varphi$,
 - $w_a = [\frac{2}{n}] \cdot 180 = [\frac{2}{n}] \pi$, The Interior angle of the [n - Polygon].
 - $w_b = [\frac{2}{n+2}] \cdot 180 = [\frac{2}{n+2}] \cdot \pi$, The Interior angle of the [$n+2$ Polygon].
 - w_o = An Extrema-angle between w_a , w_b which is related to Heights.
 - $\varphi_a = [\frac{n-2}{2 \cdot n}] \pi$, The angle of side λ_a to (e) line for Even , n -Polygon.
 - $\varphi_b = [\frac{n}{2(n+2)}] \pi$, The angle of side λ_b to (e) line for Even , $n+2$ Polygon.
 - $\varphi_o = [\frac{n-1}{2(n+1)}] \pi$, The angle of side λ_o to (e) line for Odd – Polygon .

Show that , The Extrema - angle w_o and the complementary angle φ_o , Define for the In-between Odd-Regular Pentagon in the same circle (O , OA) , and by Scanning the Δh , difference of Heights on the Circles - Heights - System , then Is succeeded on Δh , the *Harmonic Ratio of Mean on Sides* .

The Proof : Figure.-3--(2 - 3)

- a.. Draw on OK circle the Tangent (e) at Point K , and from K any two Chords KA and KB . Draw the Perpendiculars AA', BB' from Points A , B and Parallels AA₁,BB₁ to Tangent (e).
- b.. Draw the circle of Heights (A₁ , A₁B₁) In right angles triangles KAA' , KBB' , the ratios $\frac{AA'}{KA} = \frac{h_A}{\lambda_a} = \sin \varphi_a$ and $\frac{BB'}{KB} = \frac{h_B}{\lambda_b} = \sin \varphi_b$, where $h_A = \lambda_a \cdot \sin \varphi_a$ and $h_B = \lambda_b \cdot \sin \varphi_b$ and the difference $\Delta h = h_A - h_B$, or

$$\Delta h = h_A - h_B = \lambda_a \cdot \sin \varphi_a - \lambda_b \cdot \sin \varphi_b \quad \dots\dots\dots (1)$$

Since Between the two Sequent , n , $n+2$, Even – Regular – Polygons exists the Geometric logic of AB Monads , **i.e.** In a Segment the whole is equal to the Parts , and to the two halves , and for angle φ_a to become φ_b is needed to Pass through another one **Angle φ_o** , which is between the Two , **Therefore** ,

- a.. Between the Two Sequence Even-Regular-Polygons exists another One Regular-Polygon
- b.. According to Pappus theory of Proportion and Means , between the three terms h , λ , φ exists one of the three means .
- c.. Since the Sum { **it is algebraically** $n + (n+2) = 2n + 2 = 2 \cdot (n+1)$ } must be an Integer and which can be divided by 2 as exists . Therefore ,

d.. Between Any Two Numerical Even -Regular-Polygons $n, n+2$ exists The Only One $(n+1)$ Odd – Regular - Polygon .

For the commonly Divergence angle , φ , equation (1) becomes h_φ between $h_A - h_B$,

From $\Delta h = h_A - h_B = \lambda_a \sin \varphi_a - \lambda_b \cdot \sin \varphi_b$, and for the In-between is

$$\Delta h = h_A - h_B = (\lambda_a - \lambda_b) \cdot \sin \varphi = \lambda_\varphi \cdot \sin \varphi \quad \dots (2)$$

or , $h_A - h_B = (r_a \cdot \sin \varphi - r_b \cdot \sin \varphi) \cdot \sin \varphi = (r_a - r_b) \cdot \sin^2 \varphi$ **i.e.**

$$h_A - h_B = (r_a - r_b) \cdot \sin^2 \varphi = \Delta h = \lambda_\varphi \cdot \sin \varphi \quad \text{or,} \quad \frac{h_A - h_B}{\sin \varphi} = \frac{\sin \varphi}{(r_a - r_b)} = \lambda_\varphi \quad \dots (3)$$

That is , $\sin \varphi = (\frac{h_\varphi}{\lambda_\varphi})$, is the **Harmonic - Mean** between $[h_A - h_B]$, $[r_a - r_b]$

And since $[r_a] = \frac{1}{2} \sqrt{4R^2 - \lambda_a^2}$, $[r_b] = \frac{1}{2} \sqrt{4R^2 - \lambda_b^2}$, **Harmonic – Mean** is to $(\lambda_a - \lambda_b)$.

Show that , the Extrema-angle , w_o , formulates the Complementary angle , φ , defining the In-between Odd - Regular n -Polygons on the same circle (O , OK) , using the Extreme cases of this System $\{\Delta h = h_A - h_B = A_1 B_1\}$, on The Circles of difference of Height .

Analysis :

1.. From above relation of Heights and circle radius for two Sequent–Even - Polygons then

$$\Sigma h_n = n \cdot R = n \cdot OK \quad (a) \quad \text{and} \quad \Sigma h_{n+2} = (n+2) \cdot R = (n+2) \cdot OK \quad (b)$$

By Subtraction (a) , (b)

$$\Sigma h_{n+2} - \Sigma h_n = (n+2) R - n R = 2.R \quad \rightarrow \text{constant}$$

By Summation (a) , (b)

$$\Sigma h_{n+2} + \Sigma h_n = (n+2) R + n R = (n+1) . 2.R \rightarrow \text{constant}$$

i.e. In the System of Regular - Polygons the , Interior angles (w) and Gradients φ Heights (h) and their differences , Δh , – Summation and Subtraction of Heights are Interconnected and Intertwined at the Common Circle $[A, (\Delta h = h_A - h_B)]$ and thus **Producing The Common ($n+1$) , Odd – Regular - Polygon .**

2.. From Fig.-3--(2-3) $\rightarrow O_k A, O_k B$ are the Perpendicular chords on the circle which form the same angles at Point K and are $\lambda_a = 2R \cdot \sin \varphi_a$, $\lambda_b = 2R \cdot \sin \varphi_b$, and their Power - Product $[POP] = (\lambda_a \cdot \lambda_b) = 4R^2 \cdot [\sin \varphi_a \cdot \sin \varphi_b] \dots (5)$

The Sum of Heights for the n and $n+2$ Even Regular Polygon is $\Sigma h_A = n.R$ and $\Sigma h_B = (n+2).R$ and the In-between Odd Regular Polygon $\Sigma h_o = (n+1) \cdot R$.

The corresponding Interior angles are ,

$$\begin{aligned} w_a &= \left[\frac{2}{n} \right] \pi & \text{and} & \quad \varphi_a = \left[\frac{n-2}{2.n} \right] \pi & \text{or} & \quad \varphi_a = \left[\frac{\pi}{n} \right] \\ w_b &= \left[\frac{2}{n+2} \right] \pi & \text{and} & \quad \varphi_b = \left[\frac{n}{2.(n+2)} \right] \pi & \text{or} & \quad \varphi_b = \left[\frac{\pi}{n+2} \right] \\ w_o &= \left[\frac{2}{n+1} \right] \pi & \text{and} & \quad \varphi_o = \left[\frac{n-1}{2.(n+1)} \right] \pi & \text{or} & \quad \varphi_o = \left[\frac{\pi}{n+1} \right] \end{aligned}$$

The Power of Point O to the circle of diameter Δh is for $\lambda_o = 2R \cdot \sin \varphi_o$, $\lambda'_o = 2R \cdot \sin \varphi_o$ & , $[POP] = [\lambda_o \cdot \lambda'_o] = 4R^2 \cdot \sin^2 \varphi_o \dots (6)$ which is Equal to the Power from (5) and this Because the Perpendiculars from the two Centers , O . O_k , form the same Angles φ_a , φ_b

$$\text{Therefore , } \sin \varphi_a \cdot \sin \varphi_b = \sin^2 \varphi_o \quad \text{or} \quad \frac{\sin \varphi_a}{\sin \varphi_o} = \frac{\sin \varphi_b}{\sin \varphi_o} \quad \dots (7)$$

i.e. Angle φ_0 follows **The Golden Ratio Harmonic Mean of Sides** Between angles φ_a, φ_b on the $\Delta h =$ **The Golden Ratio Difference of Heights**.

3.. Since the Product of magnitudes $\lambda_a \cdot \lambda_b = \text{constant}$ and also $(\lambda_a - \lambda_b) \cdot (\lambda_a + \lambda_b)$ constant therefore, The Power of Any Point IN and OUT of the circle of Heights is also Constant, meaning that exists another one Regular – Polygon, Between the two Even - Sequence i.e.

The Outer are The Two Even - Regular n and $n+2$ Polygons, and The Inner is $\rightarrow$ The $n+1$ ODD – Regular Polygon $\leftarrow$

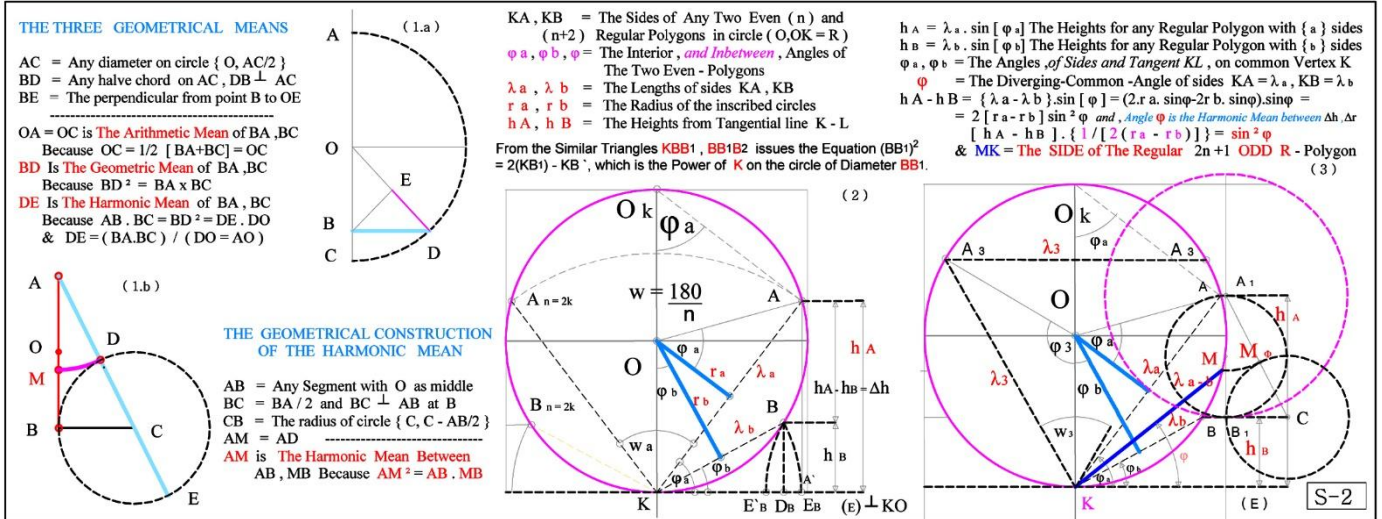


Figure-3-

In (1a) are shown the two ways for constructing the three Means on One or Two Segments .

In (1b) is The Geometrical Construction of the Golden Ratio $AM^2 = AB \cdot MB$

In (2) AB is an Arbitrary Chord on circle (O , OK = O_k) and circle (K , KB) intersects the (e) line at Point E_B . The line-Segment $KB_1 = KB$ and the Chord $BB_1 = BB_2$.

The triangles $KB E_B, B E_B E'_B$ occupy the 2-common angles $\widehat{KE_B B} = \widehat{BE'_B E_B} = \widehat{KB E_B}$

Therefore are Similar triangles and is $\frac{KB}{B E_B} = \frac{B E_B}{KB - E_B E_B}$ or $[KB]^2 - KB \cdot [E_B E'_B] = [B E_B]^2$

If we choose the Pentagon then the central angle $w_5 = 360^\circ / 5 = 72^\circ$ and $\varphi_5 = 36^\circ$.

As in the Dublication of the Cube [126] and Trisection of angle [128] , the **Right Angle Directional -Triangle** was of the relation $1 : 2 = 30^\circ / 60^\circ$, in The case of the Isosceles

Directional -Triangle $B E_B E'_B$, is needed the relation $1 : 2 = 36^\circ / 72^\circ$, where then

issues $E_B E'_B = B E_B$ and Becomes $\frac{KB}{B E_B} = \frac{B E_B}{KB - B E_B}$ or $[KB]^2 - KB \cdot [B E_B] = [B E_B]^2$.

This second degree equation is rearranged as $[B E_B]^2 + KB \cdot [B E_B] - [KB]^2 = 0$, with Two Solutions the Real and the Negative Root .

The Solution of the Real Root is $[B E_{B1}] = [\frac{KB}{2}] \cdot (\sqrt{5} - 1) = |KB| \cdot [\frac{\sqrt{5}-1}{2}] = |KB| \cdot \Phi$..(f)

Equation (f) is the Divergency of Sides to Heights of Two n , and ($n+2$) Even Polygons .

This issues in Heights because , $\sin \varphi_a = \frac{h_a}{R}$, $\sin \varphi_b = \frac{h_b}{R}$, and or Proportional to Heights .

In (3) is shown the locus of the Two - Circles on Heights (A_1, A_1B_1) and the Parallels to (e) .
to be Extrema case for the Two Segments **KA , KB** . On the circle of Diameter **A₁, B₁**,
is Applied the (1a) Method for the Harmonic Mean and Point **M** , defines on circle the
KM Chord and , the **n+1 , Odd Regular Polygon** .

b-3.. The Geometrical Construction of All The Regular Polygons :

The Analysis of the Geometrical Construction . Fig.-4--

The construction of all the Even - Regular - Polygons is Possible by dividing the circle (O ,OK)
in 2 , 4 , 6 , 8 , 10 , 12 , 14 ..2n Parts as $w_a = [\frac{2}{n}] \pi$ and $\varphi_a = [\frac{\pi}{n}]$, for $n = 1 , 2 , 3 , n$,
The construction of all the Odd - Regular -Polygons is Possible by Applying the Three Circles
System method , on the Heights of the Chords of the Even-Sequence-Sets of these known
Polygons as [2 , 4] , [4 , 6] , [6 , 8] , [8 , 10] ... [(2n)--(2n+2)] as formulas $w_o = [\frac{2}{n+1}] \pi$
 $\varphi_o = [\frac{\pi}{(n+1)}]$, and is applicable from Point K . On it is formed the Harmonic-Mean
of the Difference from the 2- Heights .

Case A → The Geometrical Construction of the **Regular - Digone , Triangle , Square , etc.**

Step 1 : Fig.-4—1-

Draw from Point K , of any circle (O , OK) , The Tangent (E) at K and the Diameter KK'
which is the Side of the Regular – Digone . The Points K , K' , are the Vertices of **R-Digone** .

Step 2 : Fig.-4-1,21-

Draw from Point K of circle (O , OK) , Circle (K ,KO) which intersects the circle at Point B_1
and Draw from Point B_1 , and from each intersection Point , the same Circle (K ,KO) , which
defines the last Point B_6 which belongs to the Regular Hexagon . By connecting the Points
K , B_2 , B_4 , K , which are the **Vertices** of **Regular - Triangle** , the between line Segments ,
 KB_2 , B_2B_4 , B_4K , are the **Sides** of the Regular -Triangle .

Step 3 : Fig.-4—1-

Draw from Point K of the circle (O , OK) , the Diameter KK' , and from the center O the
Perpendicular Diameter $a'a'$. By connecting the Points K , a , K' , a' which are the **Vertices**
of the **Regular – Tetragon** , the between line Segments , Ka , aK' , $K'a'$, $a'K$, are the
Sides of the Regular -Tetragon .

Step 4 : Fig.-4—3-

Draw from Point K of circle (O,OK) , the Circle (K ,KO) which intersects the circle (O,OK) at
Point $B \equiv B_1$ and Tangent $K(e)$ from Point K . Draw from Point B a).The circle (B, $BE_B=BE'_B$)
intersecting line $K(e)$ at Point E'_B , and b).The Perpendicular BD_B at Point D_B of $K(e)$ Tangent.

**Show that the Geometrical locus of all Points B on circle (O,OK) , on which the difference
of squares $BK^2 - BE_B^2$ from the Two constant Points K , E_B and where $KE_B = KB$, IS constant
AND is the Height BD_B at Point D_B which distances from the midpoint O_C of KE_B to the
Point D_B , AND is Equal to $\rightarrow O_C D_C = k^2 / 2.KD_B \leftarrow$ which belongs to Semi-Straight line
Sector $O_C E_B$ when $\rightarrow k^2 > 0$, And to Semi-Straight line Sector $O_C K$ when $\rightarrow k^2 < 0$.**

**For Any other Point A issue the above and also for Points A , B on the $\Delta h = h_A - h_B$.
Prove that , The Ratio on Difference Δh , Defines on KA , KB Segments The Chord
KC Between the Even , KA , KB , which is The Side of a Set-Regular–Odd-Polygon .**

THE GEOMETRICAL CONSTRUCTION OF THE ODD REGULAR - PENTAGON $n+1=5$, FROM THE TWO EVEN REGULAR $n=4$ TETRAGON & THE EVEN REGULAR $n+2=6$ HEXAGON

A.n = 4 = THE EVEN TETRAGON REGULAR POLYGON

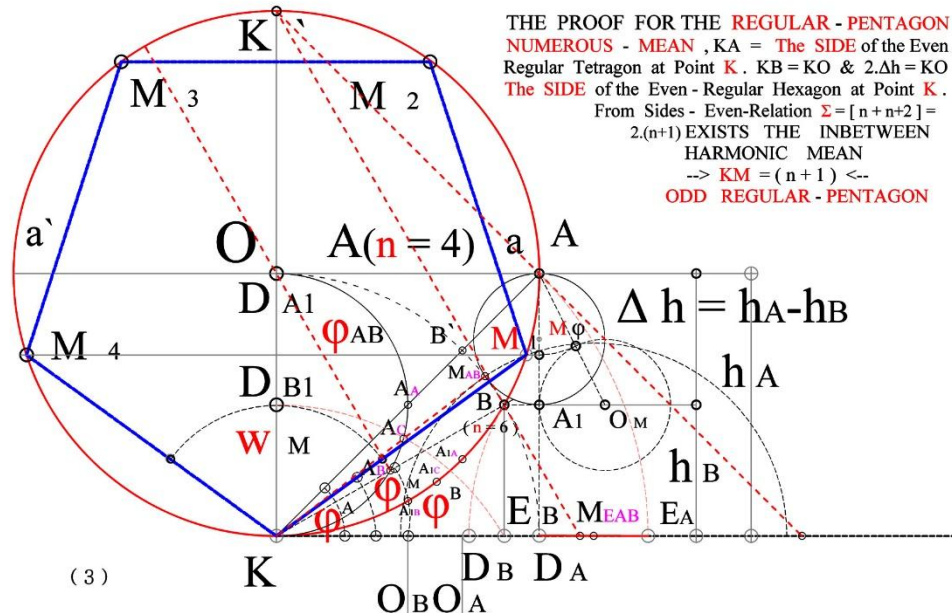
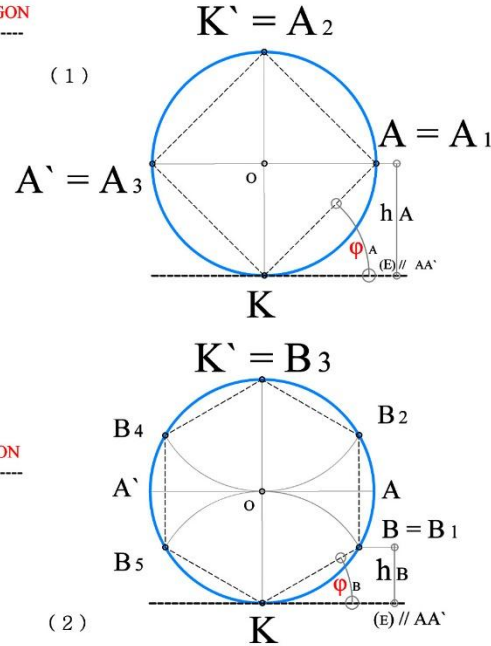
KK' = The Vertical Diameter of circle {O, KK'/2}
AA' = The Horizontal Diameter where $OA \perp KK'$
& $OA \parallel (E)$ Tangent at K.
KA = The SIDE of the Even-R.Tetragon at K.
h.A = The HIGHT of the Even-R.Tetragon on (E).
 $\phi.A$ = The ANGLE of the Even-R.Tetragon
Between the KA Side and Horizontal (E).

A.n+1 = 5 = THE ODD $n+1$, REGULAR POLYGON

KM = The SIDE of the Odd-R.Tetragon at K.
h.M = The HIGHT of the Even-R.Tetragon on (E).
 $\phi.M$ = The ANGLE of the Even-R.Tetragon
Between the KM Side and Horizontal (E).
AM = The Harmonic Mean of $\Delta h = h_A - h_B$
Because $(AM)^2 = (A_1) \times M \cdot A_1$.

A.n+2 = 6 THE EVEN HEXAGON REGULAR POLYGON

KK' = The Vertical Diameter of circle {O, KK'/2}
AA' = The Horizontal Diameter where $OA \perp KK'$
& $OA \parallel (E)$ Tangent at K.
KB = The SIDE of the Even-R.Hexagon at K.
h.B = The HIGHT of the Even-R.Hexagon on (E).
 $\phi.B$ = The ANGLE of the Even-R.Hexagon
Between the KB Side and Horizontal (E).



THE PROOF FOR THE REGULAR - PENTAGON
NUMEROUS - MEAN , KA = The SIDE of the Even
Regular Tetragon at Point K . KB = KO & $2\Delta h = KO$
The SIDE of the Even - Regular Hexagon at Point K .
From Sides - Even-Relation $\Sigma = [n + n+2] =$
 $2.(n+1)$ EXISTS THE INBTWEEN
HARMONIC MEAN
--> $KM = (n+1) <--$
ODD REGULAR - PENTAGON

Figure-4-

In (1) is shown the Geometrical Construction of the **Even Tetragon** in Circle .

In (2) is shown the Geometrical Construction of the **Even Hexagon** in Circle .

In (3) is shown the Geometrical Construction of the **Odd Pentagon** in Circle .

Analysis :

1... The Geometrical Construction of the Regular Tetragon and Hexagon is in Fig.4--1,2-

2... The Geometrical Construction of the Regular Pentagon is as the Step-4 & Fig.4--3--

3... To Prove that Triangle $\mathbf{KBE_B}$ is the **Directional-Triangle of The Regular-Polygons** .

For the Point $\mathbf{B}$ of circle $(\mathbf{K}, \mathbf{KB} = \mathbf{KO})$ issues $\mathbf{BK}^2 - \mathbf{BE_B}^2 = \mathbf{KD_B}^2 + \mathbf{BD_B}^2 - \mathbf{BD_B}^2 - \mathbf{D_BE_B}^2$ and $[\mathbf{BK}^2 - \mathbf{BE_B}^2] = \mathbf{KD_B}^2 - \mathbf{D_BE_B}^2 \dots\dots(\mathbf{a})$. From Pythagorean 2nd Theorem of Diamesus on the Isosceles Triangle $\mathbf{KBE_B}$ issues $[\mathbf{BK}^2 - \mathbf{BE_B}^2] = 2 \cdot \mathbf{KE_B} \cdot [\mathbf{O_BD_B}] \dots\dots\dots(\mathbf{b})$

where $\mathbf{O_B}$ is the midpoint of $\mathbf{KE_B} = \mathbf{KB}$. Rearranging the equation (b) then ,

$$[\mathbf{O_BD_B}] = [\mathbf{BK}^2 - \mathbf{BE_B}^2] / 2 \cdot [\mathbf{KE_B} = \mathbf{KB}] == [\mathbf{KD_B}^2 - \mathbf{D_BE_B}^2] / 2 \cdot [\mathbf{KB}] \dots\dots(\mathbf{c})$$

i.e. The Distance $[\mathbf{O_BD_B}] = \mathbf{k}^2 = \mathbf{A}$ Constant $\mathbf{BD_B}$ at Foot-Height Point $\mathbf{D_B}$.

Since the Points $\mathbf{K}, \mathbf{B}, \mathbf{E_B}, \mathbf{D_B}, \mathbf{O_B}$, are **Fix & Constants** for Triangle $\mathbf{KBE_B}$, then $\rightarrow$

The Geometrical locus of Points $\mathbf{E_B}$, which difference of squares $\mathbf{BK}^2 - \mathbf{BE_B}^2 = \mathbf{k}^2$ from the Two constant Points $\mathbf{K}, \mathbf{E_B}$, is the Constant Height $\mathbf{h_B}$ equal to $|\mathbf{BD_B}|$ at the Constant Point $\mathbf{D_B}$ which Distances from the midpoint $\mathbf{O_B}$ of $\mathbf{KE_B}$ as

$\rightarrow [\mathbf{O_BD_B}] = \mathbf{k}^2 / 2 \cdot \mathbf{KE_B} = \mathbf{constant} \leftarrow$ In the same way for an Point $\mathbf{A}$ issues ,

The Geometrical locus of Points $\mathbf{E_A}$, which difference of squares $\mathbf{AK}^2 - \mathbf{AE_A}^2 = \mathbf{k}^2$ from the Two constant Points $\mathbf{K}, \mathbf{E_A}$, is the Constant Height $\mathbf{h_A}$ equal to $|\mathbf{AD_A}|$ at the Constant Point $\mathbf{D_A}$ which Distances from the midpoint $\mathbf{O_A}$ of $\mathbf{KE_A}$ as

$\rightarrow [\mathbf{O_AD_A}] = \mathbf{k}^2 / 2 \cdot \mathbf{KE_A} = \mathbf{constant} \leftarrow (\mathbf{d}) \leftarrow$

On Right angle Triangle $\mathbf{KAK'}$ issues $|\mathbf{KA}|^2 = |\mathbf{KD_{A1}}|^2$. $\mathbf{KK'} = \mathbf{h_A} \cdot 2\mathbf{R}$ and $|\mathbf{KB}|^2 = \mathbf{h_B} \cdot 2\mathbf{R}$,

By Subtraction $|\mathbf{KA}|^2 - |\mathbf{KB}|^2 = 2\mathbf{R} \cdot [\mathbf{h_A} - \mathbf{h_B}]$ or $\Delta\mathbf{h} = \mathbf{h_A} - \mathbf{h_B} = [\mathbf{KA}^2 - \mathbf{KB}^2] / 2\mathbf{R} \dots\dots(\mathbf{e})$

i.e. **The Geometrical locus of any Two-Even-Sequence-Sets of Points $\mathbf{A}, \mathbf{B}$, is On their constant Difference of Heights $\Delta\mathbf{h} = \mathbf{h_A} - \mathbf{h_B}$. (Conclusion-1) And from ..(d)**

3a... When the constant difference $[\mathbf{O_AD_A}] = \mathbf{k}^2 / 2 \cdot \mathbf{KE_A} = \mathbf{AK}^2 - \mathbf{AE_A}^2 > 0$, Then $[\mathbf{O_AD_A}]$ Belongs to the Half-Distance $\mathbf{O_AE_A}$, which is the corresponding **Index of Chord $\mathbf{KA}$.**

3b... When the constant difference $[\mathbf{O_AD_A}] = \mathbf{k}^2 / 2 \cdot \mathbf{KE_A} = \mathbf{AK}^2 - \mathbf{AE_A}^2 < 0$, Then $[\mathbf{O_AD_A}]$ Belongs to the Half-Distance $\mathbf{O_AK}$, which is the corresponding **Index of Chord $\mathbf{KA}$.**

3c... When the constant difference $[\mathbf{O_AD_A}] = \mathbf{k}^2 / 2 \cdot \mathbf{KE_A} = \mathbf{AK}^2 - \mathbf{AE_A}^2 \equiv 0$, Then $[\mathbf{O_AD_A}]$ Belongs to the Point $\mathbf{D_A}$ which is an **Indeterminacy of Point $\mathbf{A}$ and Chord $\mathbf{KA}$ at $\mathbf{D_A}$.**

3d... For a Chord $\mathbf{KX}$, Between Any Even - Sequence-Set , $\mathbf{KA} - \mathbf{KB}$, and when issues $\mathbf{KX} = \mathbf{KE_X} \equiv \mathbf{KD_A}$, then **Indeterminacy is transferred to Pole $\mathbf{D_A}$ where $|\mathbf{XK}^2 - \mathbf{XD_A}^2| = 2 \cdot |\mathbf{KD_A} \cdot \mathbf{O_XD_X}|$ & $\rightarrow [\mathbf{O_XD_X}] = |\mathbf{XK}^2 - \mathbf{XD_A}^2| / 2 \cdot \mathbf{KD_A} = \mathbf{Constant} \leftarrow$ (Conclusion-2)**

3e... For The Chord $\mathbf{KX}$, Between The Even - Sequence-Set [4-6] , which is the Regular Tetragon with $|\mathbf{KA}| \equiv |\mathbf{Ka}| = \mathbf{R}\sqrt{2}$ and Regular Hexagon with $|\mathbf{KB}| \equiv \mathbf{KD_A} = |\mathbf{R}|$ and $\mathbf{BD_A}$ Rotates to $\mathbf{BE_b}$ to Construct $\mathbf{KBE_b}$ from where issues $\mathbf{KE_b} = \mathbf{KD_A} = \mathbf{R}$, and from (b) $[\mathbf{O_BD_B}] = [\mathbf{BK}^2 - \mathbf{BE_B}^2] / 2 \cdot \mathbf{KE_B} \rightarrow [\mathbf{BK}^2 - \mathbf{BD_A}^2] / 2 \cdot \mathbf{BK}$, Or $[\mathbf{BK}^2 - \mathbf{BE_B}^2] = 2 \cdot \mathbf{KE_B} \cdot [\mathbf{O_BD_B}] \rightarrow [\mathbf{BK}^2 - \mathbf{BD_A}^2] = 2|\mathbf{BK}| \cdot [\mathbf{O_BD_B}] \dots\dots\dots(\mathbf{g})$ i.e.

Since the Points K, B, E_B, D_B, O_B ARE **Fix & Constant** for Triangle $KBE_B \equiv KBD_A$, and **The Geometrical locus of Points E_B Exist also in the Prior KBD_A** Therefore In Even - Sequence-Set [4 - 6], Triangle KBD_A is **Fix & Constant**. In the same Locus AD_A and from the Side of the Regular $n = 4$ -Polygon Exists and is Calculated the Side of the Regular $2n = 8$ -Polygon, and in the Case of Inbetween, $n+1 = 5$ Regular Pentagon Exists and is Calculated the Side of the Regular $2 \cdot |n+1| = 10$ Regular Decagon. At the same time,

The Geometrical locus of Points E_A , which difference of squares $AK^2 - AE_A^2 = k^2$ from the Two constant Points K, E_A , is the Constant Height h_A equal to $|AD_A|$ at the Constant Point D_A , which Distances from the midpoint O_A of KE_A are as

$$\rightarrow [O_B D_B] = k^2 / 2. KD_A = [BK^2 - BD_A^2] / 2. KD_A = \text{constant} \leftarrow$$

3f...For Point C of circle (O, OK) where Angle φ_C exists on Chord KC , issues

$$\sin \varphi_C = \frac{KC}{2.R}, \& \left| \frac{KB}{\sin \varphi_B} \right| = 2.R, \left| \frac{KC}{\sin \varphi_C} \right| = 2.R, \text{ and } \left| \frac{KA}{\sin \varphi_A} \right| = 2.R = \text{Constant},$$

$$\text{From (c)} \quad h_A - h_B = \frac{|KA|^2 - |KB|^2}{2.R}, \quad h_A - h_C = \frac{|KA|^2 - |KC|^2}{2.R}, \quad h_C - h_B = \frac{|KC|^2 - |KB|^2}{2.R},$$

$$\text{By division } [h_A - h_C]^2 = \frac{|KA|^2 - |KC|^2}{|KA|^2 - |KB|^2 \cdot |KC|^2 - |KB|^2} [h_A - h_B] \cdot [h_C - h_B] = \frac{|KA|^2 - |KC|^2}{R \cdot |KC|^2 - |KB|^2} [h_C - h_B]$$

And this Because $h_A - h_B = R - R/2 = R/2$ and $|KA|^2 - |KB|^2 = |R\sqrt{2}|^2 - R^2 = R^2 \dots \therefore$

The above Relation Denotes the Correlation of **Heights and Sides** and needs a further

4... The Apothem Relations. Fig.4--3--

Apothem is the Distance of Center O to the Midpoint of Sector KB , and to any other Sector KA . When Apothem is extended to the Circle of Diameter KO Intersects this circle at Point A_B , i.e. Point A_B is on The Locus of Apothems {*The Circle of Diameter KO* } & according to Archimedes, **Point A_{1B} is the Vertex of the Regular $2v$ Regular Polygon on The Circle of Diameter KO** . (Conclusion-3)

5...On The Isosceles Twin-Couple Triangle KBE_B issues, $|BK|^2 - |BE_B|^2 = 2 \cdot |BK| \cdot |O_B D_B|$

Which is a Constant, and the same for Triangle KAE_A , $|AK|^2 - |AE_A|^2 = 2 \cdot |AK| \cdot |O_A D_A|$ For Any **Two-Even-Sequence-Sets**, $n - n+2$, of Points A, B , of circle (O, OK) and Sides KA, KB with Difference of Heights $\Delta h = h_A - h_B$, or Projections, And Angle $\widehat{A \tilde{K} B}$ is the Difference of Angles $\varphi_A - \varphi_B = \varphi_{AKB} = \widehat{A \tilde{K} B}$. The Corresponding Foots - Difference of the Heights $[D_A D_B]$ is **Equal** to that of Apothems $K.A_A - K.A_B$, and this Because correspond to the same Angle φ_{AKB} . (Conclusion-4)

Extending KA_A, KA_B on $[K, KB]$ circle where $|KB| = 2 \cdot |KA_B|$, then $K'B$ is the Tangent at Point B which is line BK' . The same issues for KA_A , $|KA| = 2 \cdot |KA_A|$, & $K'A$ is the Tangent at Point A which is line AK' . (Conclusion-5)

6...On Circle $[K, KB]$, All Points occupy the Property of **Constant Sum Squares** from the two Constant Points E_A, E_B , as, $|AK|^2 + |AE_A|^2 = \text{Constant}$, $|BK|^2 + |BE_A|^2 = \text{Constant}$. For Middle Point M_{AB} of Arc $\widehat{AB}$ issues also $|M_{AB}K|^2 + |M_{AB}E_A|^2 = \text{Constant}$, (Conclusion-6).

**b-4.. The 3-Geometrical Methods for Constructing ALL the Regular Polygons :
Using the Coupler Curves [63] 16 / 06 / 2017 .**

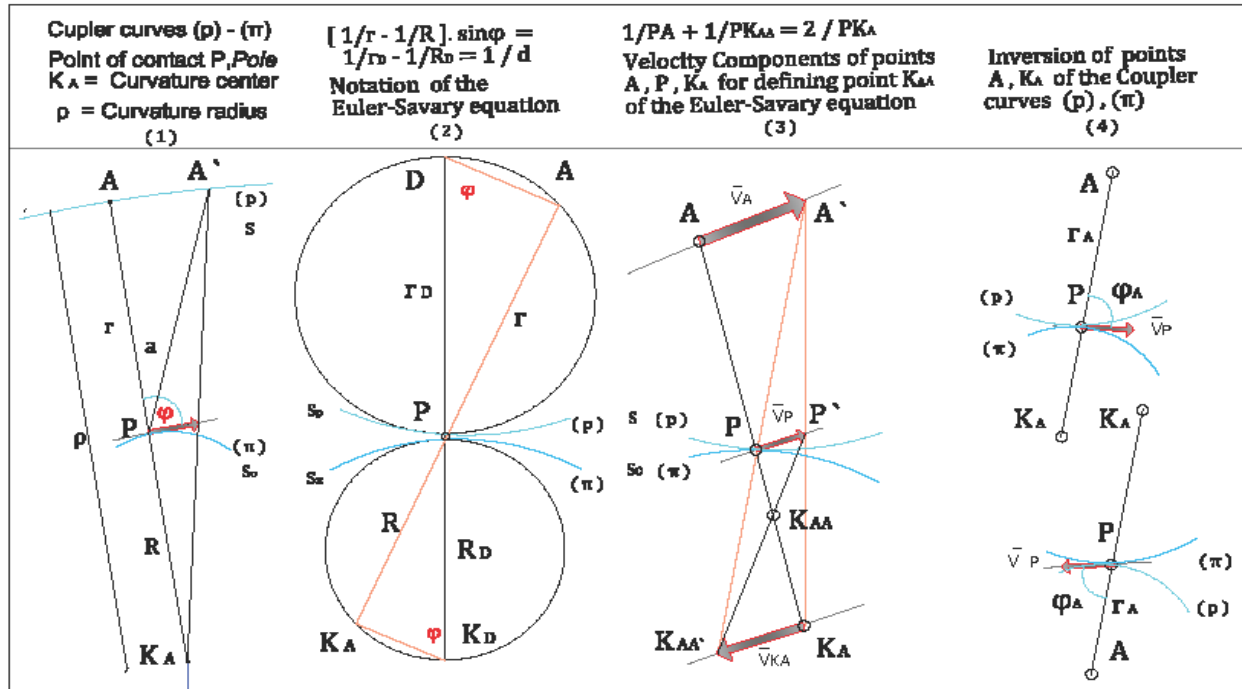


Figure-5-

a.. The [Antiphon – Archimedes] Ancient Greek – Polygons method .

b.. The [Euler - Savary] Coupler-Curves curvature - centers method .

c.. The [Markos Geometrical , Three – Circles - Method , in Polygons .

In (1) A Point A on Coupler-curves (p) , (π) define the Point of curvature K_A , the Instantaneous Point P , the Pole on (π) .

In (2) is the case of Point A lying on radius of curvature of Polar Path (Point D) where then the Paths of Points A in S , System have the Instantaneous center of curvature K_A on the fixed System S_0 .

In (3) The Velocity Instantaneous center for curvature Point K_A , in S_0 System is Point K_{AA}

In (4) The two Points A , K_A , of Coupler-curves (p) , (π) , follow the inversed motion where the two Poles of rotation , A and K_A , are inverted .

C... THE TWIN – COUPLE TRIANGLES Method FOR THE CONSTRUCTION OF THE REGULAR --- Odd --- POLYGONS .

Data : Fig.-6-1,2,3,4-

The circles and the Three Triangles are constructed using a Ruler and a Compass .

- 1... In (1) The Construction of the Golden Ratio **DM** on the Straight line Segment **KD** where $DM = \Phi \cdot DM$ and , $DM^2 = DM \cdot MK$,
- 2... In (2) The Construction of the Regular Tetragon with Side **KA'** , and Hexagon with Side **KB = KO**
- 3... In (3) The Construction of the Sides in a Regular ODD Polygons **A₁ A_{k+1}** , **A₁ A_{k+2}** , with Their relation on Polygon Heights h_{k+1} , h_{k+2} .
- 4... In (4) and on the two circles [O,OK] , [K,KA] , are Drawn the Three Twin-Coupled Triangles , **KAK'** , **KAE_A** , **KAE_a** , which solve the Odd - Problem .

C1.. The Construction of The Regular Polygone And Pentagone : Fig -6—4-

STEP-1 $\Rightarrow$ Draw the circle (O , OK) and the two Perpendicular Diameters , **aa' $\perp$ KK'** .

And the Tangent **K(E)** at Point **K** , **Using a Ruler and a Compass.**

STEP-2 $\Rightarrow$ Draw **KA** , **KB** the Two Segments such that are the Sides of an , **n = 4 & n+2 = 6** Even Regular Polygone Tetragone , Hexagone , **Using a Ruler and a Compass.**

STEP-3 $\Rightarrow$ Draw circles (K, KA) , (K, KB) which intersect tangent **K(E)** at Points **E_A** , **E_B** , **And** the line-Sectors **AE_A** , **BE_B** . respectively , **Using a Ruler and a Compass.**

STEP-4 $\Rightarrow$ Draw the Perpendiculars **AD_A** , **BD_B** on Tangent **K(E)** , **And** the two circles with **Diameters** , **KA** , **KB** , **Using a Ruler and a Compass.**

STEP-5 $\Rightarrow$ Draw the circle with **Diameter KB** , to Intersect the Sectors KA , KB at Points **A_A** , **A_B** , Apothems **OA_A** , **OA_B** , Produced at Points **A_{1A}** , **A_{1B}** , of Circle (K , KB) and Apothem **A_C** , from Mid-arc **A_{1C}** , & Points **B** , **B'** , **Using a Ruler and Compass.**

STEP-6 $\Rightarrow$ Draw the line-Sectors **OA_A** , **OA_B** and **K'A** , **K'B** respectively , and Project the line-Sector **KA_C** to circle (K , KB) , from Midpoint **A_C** to Midpoint **M_{AB}** **Using a Ruler and a Compass .**

STEP-7 $\Rightarrow$ Draw The circle (**M_{EAB}** , **M_{EAB} M_{AB}**) with Radius [**M_{EAB} M_{AB}**] and center The Midpoint **M_{EAB}** , of Couple Points **E_A** , **E_B** , **Such that to Intersect the Circle** (O , OK) at Point **F_{C = n+1}** , **Using a Ruler and a Compass .**

{ Chord KF_{C = n+1} = KC is the Odd Regular Side of The IN-Between Even-Set }

To Prove the followings :

- a)...That Between Any **Two-Even-Sequence-Sets** of Points , A , B , of circle (O, OK) and Sides KA , KB , **EXISTS** a Chord **MC** which is **The Side of their In-Between Regular - Odd- Polygon** . with Difference of Heights $\Delta h = h_A - h_B$, or of Projections
- b)... That The Geometrical locus of Any Two Even-Sequence-Sets of Points A , B , is Between their **Directional-Triangle** , from the constant Difference of their Height Projections , and On the Apothem Circle .

c)... For The Geometrical Construction of the Regular Polygons is needed the Couple of Triangles KAK' , $KA.E_A$, $KB.E_B$ and the combined **Circle of Apothems** with Diameter KO such for n and $2n$ Polygons, and the circle of **Constant Sum Squares** (M_{EAB} , M_{EAB} , M_{AB}) combined with the main circle (O , OK).

d)... That The Golden Ratio on the Height Difference $D_{A1} - D_{B1} = \Delta h = h_A - h_B$, issues Also **AND** for the Regular Pentagon - Decagon Only.

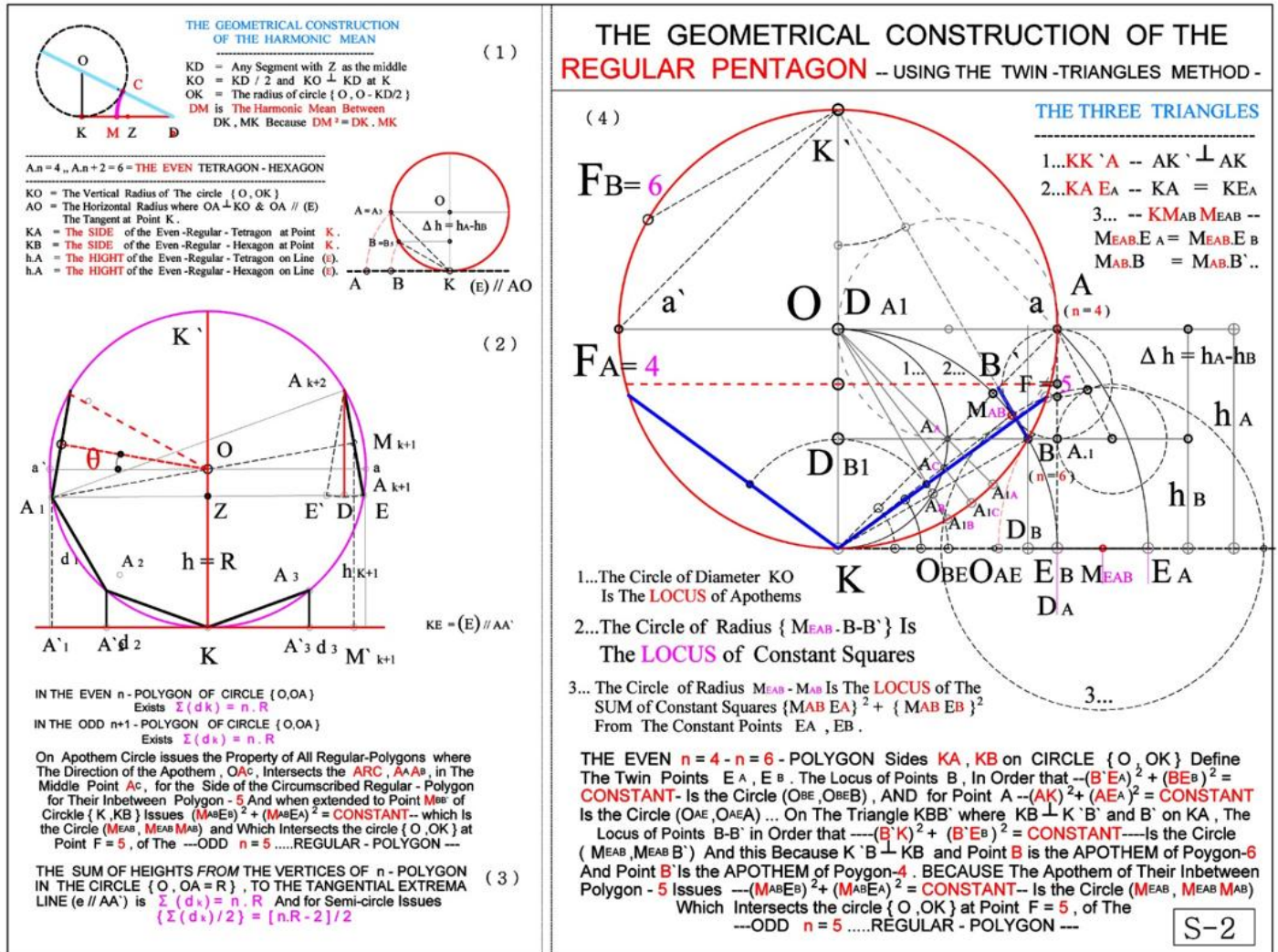


Figure-6-

In (1) is shown the Geometrical Construction of The Golden Ratio on a Difference $DK = \Delta h$.
 In (2) is shown the Geometrical Construction of Tetragon = n and Hexagon = $n + 2$ in (O, OK) .
 In (3) is shown the Geometrical Proof of the $n = 2.k$ Even and $n = 2k+1$ Odd Polygons.
 In (4) is shown $\rightarrow$ **The Directional - Triangles $KA.E_A$, $KB.E_B$, The Circle of Apothems** (DB_1 , $DB_1K = DB_1O$), **The Circle** (K , $|KB| = |KB'|$), of **Constant Different Squares** **The Circle of Constant Sum Squares** (M_{EAB} , M_{EAB} , M_{AB}) and All Points on the Main circle (O , OK) and axis.

C2.. THE NEEDED PROOFS :

-(a)- Since the **Algebraical Sum** $\{ n + (n+2) = 2n + 2 = 2.(n+1) \}$ must be an Integer and which also can be divided by 2 as this exists. **Therefore**,
Between Any Two Numerical Even - Regular-Polygons $n, n+2$ exists The Only One $(n+1)$ Odd – Regular - Polygon .

-(b)- From Relation of Heights and circle radius for **two Sequent–Even- Sets** of Polygons ,
 $\Sigma h_n = n . R = n . OK$, and $\Sigma h_{n+2} = (n+2) . R = (n+2) . OK$, *By Subtraction*
 $\Sigma h_{n+2} - \Sigma h_n = (n+2) R - n R = 2.R \rightarrow \text{constant}$
And By Summation ,
 $\Sigma h_{n+2} + \Sigma h_n = (n+2) R + n R = (n+1) . 2.R \rightarrow \text{constant}$

Therefore , The constant Difference of Heights exists also On Directional-Triangles .

-(c)- The construction of the Two Sequent–Even- Sets is Possible by dividing the circle (O,OK) in 2, 4, 6, 8, 10, 12, 14 ..2n Parts as $w_a = [\frac{2}{n}] \pi$ & $\varphi_a = [\frac{\pi}{n}]$, for $n = 1, 2, 3, n$
From these known Polygon-Sets [2, 4], [4, 6], [6, 8], [8, 10]...[(2n) -(2n+2)] as the n-Side= **KA** Sector and n+2 Side= **KB** Sector ,To n+1Side= **KC** as $w_c = [\frac{2}{n+1}] \pi$ $\varphi_c = [\frac{\pi}{(n+1)}]$

And is Applicable from Point K , To the Circle of Apothems , A_A, A_B, A_C , To the Circle of the Constant Sum Squares at , B , B' , M_{AB} , **Combined** to the Couple Triangles KA. E_A , KB. E_B , with Constant Difference Squares from Points . E_A, E_B .

-(d)- **THE PROOF FOR THE SUM OF HEIGHTS :**

Figure.-6--3-

-(a)- The Line , $A_1 A_{k+1}$, is Parallel to K-(e) line and $\perp KO$ therefore $KA_1 = KA_{k+1}$.
The mid-Perpendicular OM_{k+1} Passes from A_1 Vertex , therefor the Straight line-Segment $A_1 A_{k+1} = A_1 A_{k+2}$. The Segment $A_1 M_{k+1} = A_1 O + OM_{k+1} = R + r_{k+1}$
In Trapezium $A_1 M_{k+1} M'_{k+1} A'_1$, which is on the Vertice A_1 and with the Bases $A_1 A'_1$ and $M_{k+1} M'_{k+1} = h_{k+1}$, the Distance $KO = h$ is Parallel to Bases and divides the $A_1 M_{k+1}$ Segment into the Parts R , r_{k+1} . From Trapezium- Lemma $\rightarrow$

$$\text{Distance } \mathbf{KO} = \mathbf{h} = \left[\frac{R.h_{k+1} + r_{2k+1} . d_1}{R + r_{2k+1}} \right] \dots\dots (1)$$

Distance KO is Common to all $2k+1$ Trapeziums as that on Vertex A_1 , and keeps the Ratio $[R / r_{2k+1}] = \text{Constant} \dots\dots (2)$. By adding all the homonymous Distances of (1) which all give $KO = h$ from the Between them Different Trapeziums then issues ,

$$\begin{aligned} [2k+1].h &= \left[\frac{R[h_{k+1} + h_{k+2} + h_{2k+1} + h_1 + h_k] + r_{2k+1} . [d_1 + d_2 + d_{k+1} + d_{2k+1}]}{R + r_{2k+1}} \right] = \left[\frac{R.S h_i + r_{2k+1} . \Sigma d_i}{R + r_{2k+1}} \right] \\ &= [2k+1].h = (n+1).h = \left[\frac{R.S h_i + r_{2k+1} . \Sigma d_i}{R + r_{2k+1}} \right] = \frac{R.S + r_n \Sigma}{R + r_n} \dots\dots (3) \end{aligned}$$

Where $S = h_1 + h_2 + \dots h_{2k+1}$, and since $h_1 = (d_1 + d_2) / 2$, then S becomes ,

$$S = h_1 + h_2 + \dots h_{2k+1} = \frac{1}{2} [(d_1 + d_2) + (d_2 + d_3) + \dots (d_{2k} + d_1)] = \frac{1}{2} [2(d_1 + d_2 + \dots d_{2k+1})] = \Sigma$$

$$\text{And the above equation becomes } (n+1).h = \frac{R.S + r_n \Sigma}{R + r_n} = \frac{[R + r_n] . \Sigma}{R + r_n} = \Sigma = (n+1).h \dots(4)$$

i.e. a).. For any n-Even Regular Polygon and (n+1) Odd Regular Polygon ,The Summation Σ The Heights is equal to $\rightarrow n . KO = n . R \leftarrow$ for Number n of the Polygon.

b).. For an Even n - Polygon exist n Apothems r_n with $2.n$ Trapeziums ,
 For an Even $n+2$ - Polygon exist $n+2$ Apothems r_{n+2} with $2.n+2$ Trapeziums ,
 From above **n-gone-difference** $-n+2 - n = 2$ - Apothems $r_{n+2} - r_n = 2$, Trapeziums $2n+2 - 2n = 2$
 Or , **The In-between Odd Polygons are Independent of the Even - Sequence – Sets .**

And Their Side λ_{n+1} also , are in their In-Between Even Sequence Sets .

In Figure Fig.-6—3 , **The Twin-Couple of Triangles** $K A_{k+1} A_{k+2}$ & $A_{k+2} A_{k+1} E$, where
 $DE = DA_{k+1}$ and the Side , $K' A_{k+1}$, of the $2-n$ Polygon Parallel to $A_{k+2} D$.

The General Pythagorean theorem for Side $A_{k+1} A_{k+2} = A_{k+1} E$ gives ,

$$\begin{aligned} [A_{k+1} A_{k+2}]^2 &= [E A_{k+2}]^2 = [K A_{k+2}]^2 + K E^2 - 2.[K E] & , \text{ or} \\ [A_{k+1} A_{k+2}]^2 &= [K A_{k+2}]^2 - [K E] * \{ 2.[K D] - K E \} & , \text{ or} \\ [A_{k+1} A_{k+2}]^2 &= [K A_{k+2}]^2 - [K E] * \{ K D \} + D E \} & , \text{ or} \\ [A_{k+1} A_{k+2}]^2 &= [K A_{k+2}]^2 - [K E] * \{ K D \} + D A_{k+1} \} & , \text{ or} \\ [A_{k+1} A_{k+2}]^2 &= [K A_{k+2}]^2 - \{ K A_{k+1} * K E \} \dots\dots(5) & , \text{ i.e.} \end{aligned}$$

The Straight - line Sector $[K E]$, controls The Construction of The Odd Polygons .

On the Isosceles Triangles KAE_A , KBE_B , exist $KA = KE_A = \lambda_A$ & $KB = KE_B = \lambda_B$,
 The $\sin \varphi_A = \frac{h_A}{\lambda_A} = \frac{\lambda_A}{2R_A}$ because $K'A \perp KA$ and , $\sin \varphi_B = \frac{h_B}{\lambda_B} = \frac{\lambda_B}{2R_B}$ because $K'B \perp KB$.
 From above $[\lambda_A]^2 = 2R_A \cdot h_A$ and , $[\lambda_B]^2 = 2R_B \cdot h_B \dots\dots(1)$
 Applying above relation to the **Even Sequence Sets** then for **their Odd KC** , issues ,

$$\sin \varphi_A = \frac{KA}{2.R_A} = \frac{\lambda_A}{2.R_A} = \frac{h_A}{\lambda_A} \text{ and then } [\lambda_A]^2 = 2R_A \cdot h_A \dots\dots(2a)$$

$$\sin \varphi_B = \frac{KB}{2.R_B} = \frac{\lambda_B}{2.R_B} = \frac{h_B}{\lambda_B} \text{ and then } [\lambda_B]^2 = 2R_B \cdot h_B \dots\dots(2b)$$

$$\sin \varphi_C = \frac{KC}{2.R_C} = \frac{\lambda_C}{2.R_C} = \frac{h_C}{\lambda_C} \text{ and then } [\lambda_C]^2 = 2R_C \cdot h_C \dots\dots(2c)$$

From $\Delta h = h_A - h_B = KA \cdot \sin \varphi_A - KB \cdot \sin \varphi_B$, and for their In-between Index
 is $\Delta h = h_A - h_B = (KA - KB) \cdot \sin \varphi = AB \cdot \sin \varphi$, or
 $h_A - h_B = (r_A \cdot \sin \varphi - r_B \cdot \sin \varphi) \cdot \sin \varphi = (r_A - r_B) \cdot \sin^2 \varphi$ and
 Placing the difference of Heights $\Delta h = h_A - h_B$, then

$$h_A - h_B = (r_A - r_B) \cdot \sin^2 \varphi = \Delta h = KC \cdot \sin \varphi \text{ or , } \frac{h_A - h_B}{\sin \varphi} = \frac{\sin \varphi}{(r_A - r_B)} \dots\dots(\varphi)$$

i.e. $\rightarrow \sin \varphi = (\frac{C.D_C}{KC}) \leftarrow$ is the Harmonic - Mean Between $[\Delta h]$, $[r_A - r_B]$

Placing the Difference of Heights on **KO Axis** then , $\Delta h = h_A - h_B = D_{A1} D_{B1} \dots\dots(\varphi 1)$

-(d)- In Fig.-6-4 , For Regular Hexagon issues $D_{A1} D_{B1} = R - R/2 = R/2 = D_{B1} A_A = \Delta h$

And the Projection of Point A on $D_{B1} A_A$ axis is $A_A A_1 = AA_1 = A_A D_{B1} = R/2$, or

$$h_A - h_B = R - R/2 = R/2 , \text{ and } r_B - r_A = \sqrt{3} \cdot |R/2| - |R/2| = |R/2| \cdot |\sqrt{3} - 1|$$

Since The Harmonic-Mean which issues on Radius $AO = AD_A$, issues also on Halve Radius AA_1 Analogically on BD_{B1} Sector and at D_{B1} Foot of Apothem for Decagon .

These Points are the Constant Point-Positions on the two Axis , **Therefore** ,

The Projection D_C of the ODD In-Between Polygon follows (φ) , where Angle φ Is the Mean of difference of Heights $h_A - h_B$, to CD_C Pole line .

-(e)- THE PROOF FOR The Directional-Triangles KA.E_A , KB.E_B ,

Figure.--6—4—

-(e1)- From the Pythagoras theorem On Right-angle-Triangle [K,AK'] issues

$$(AK)^2 = (KK')^2 - (AK')^2 = (2R)^2 - (AK')^2 = 4R^2 - (2R \cos \varphi_A)^2 = 4R^2 - 4R^2 \cdot \cos^2 \varphi_A = 4R^2 [1 - \cos^2 \varphi_A] = 4R^2 \cdot \sin^2 \varphi_A = (KA)^2 \dots (1) \text{ or}$$

$$\sin \varphi_A = \frac{KA}{2.R} \text{ ,, } \sin^2 \varphi_A = \left| \frac{KA}{2.R} \right|^2 \text{ ,, } \left| \frac{KA}{\sin \varphi} \right|^2 = 2.R, \left| \frac{KA}{\sin \varphi} \right|^2 = 4.R^2 = \text{Constant} ,$$

-(e2)-Applying (1) on the Isosceles Triangle KAE_A , then $\sin^2 \varphi_A = \left| \frac{KA}{2.R} \right|^2 = \left| \frac{(AD_A)^2}{(KA)^2} \right| = \left| \frac{AD_A^2}{2R.h_A} \right|$

or , $\sin^2 \varphi_A = \left| \frac{AD_A^2}{2R.h_A} \right| = [KA^2 / 4R^2]$ and rearranging $\left| \frac{KA^2}{AD_A^2} \right| = \left| \frac{2R}{h_A} \right| = \left| \frac{2R}{R} \right| = 2 \dots (2)$ Besides

$\varphi_A = 180^\circ - 2 \cdot \varphi_a$ and then $2 \cdot \varphi_a = 180^\circ - \varphi_A$, **And** $\sin 2 \cdot \varphi_a = 2 \cdot \sin \varphi_a \cdot \cos \varphi_a$. **i.e.**

Issues on the Isosceles Triangle $\rightarrow \left| \frac{AD_A}{AE_A} \right| = 2 \cdot \left| \frac{AD_A}{KA} \right| \cdot \left| \frac{KD_A}{KA} \right| \leftarrow$ OR $|KA|^2 = 2 \cdot |KD_A| \cdot |AE_A|$

The same for KB , KC $\rightarrow |KB|^2 = 2 \cdot |KD_B| \cdot |BE_B|$ & $|KC|^2 = 2 \cdot |KD_C| \cdot |CE_C| \leftarrow \dots (3)$

-(e3)- Applying (5) On Isosceles Triangle [AKE_A] then $|AK|^2 - |AE_A|^2 = 2 \cdot |KD_A - D_A E_A| \cdot |KE_A|$

and if O_A is the middle of KE_A then issues $|KA|^2 - |AE_A|^2 = 2|KA| \cdot |O_A D_A|$ or ,

$$2 \cdot |KA| \cdot |O_A D_A| , \text{ and issues } \rightarrow |AK|^2 - |AE_A|^2 = 2 \cdot |AK| \cdot |O_A D_A| \leftarrow \dots (4)$$

i.e. The Difference of the Squares $|AK|^2 - |AE_A|^2$ on the Isosceles Triangle KAE_A , KA=KE_A from Point A , to the Two Constant Points K , E_A , is Constant and Equal to the Product of the Double Chord KA to the Distance of Midpoint to the Foot of A . The same for B The Difference of the Squares $|BK|^2 - |BE_B|^2$ on the Isosceles Triangle KBE_B , KB=KE_B from Point B , to the Two Constant Points K , E_B , is Constant and Equal to the Product of Twice the Chord KA multiplied by the Length between the Midpoint O_A and the Foot D_A of Point A . **Therefore ,**

The Difference of Squares is Constant for all Even-Sequence-Sets Chords .

-(f)- THE PROOF FOR CONSTANT Sum Squares in The Directional-Triangles .

By applying the Median Theorem for Triangles to the Directional Triangles then is Revealed the Locus of the Sum Squares .

On Triangle KBE_B , issues $|BK|^2 + |BE_B|^2 = 2 \cdot |O_{BE}B|^2 + \frac{|KE_B|^2}{2} = \text{Constant-s}^2$, and since

KE_B = |KB| and $|O_{BE}B| = |KB| / 2$, then Becomes $\rightarrow |O_{BE}B|^2 = s^2 / 2 - \frac{|KE_B|^2}{4} \leftarrow$ or

$$\text{Radius } |O_{BE}B| = \sqrt{s^2 / 2 - \frac{|KE_B|^2}{4}} = \frac{1}{2} \sqrt{2 \cdot s^2 - |KB|^2} = \text{Constant} \dots (f-1) \text{ i.e.}$$

Point B on Circle |O_{BE} , O_{BE} B| formulates Triangle KBE_B , such that The Squares from B To the Two Constant Points K , E_B is Constant as (f-1) &

By Changing the Two Constant Points to E_B , E_A , their New Sum is ,

$$[BE_A|^2 + BE_B|^2] = s^2 , \text{ and their Center } O_{BE} \text{ is their New Midpoint } M_{EAB} ,$$

Therefore , All Points of this Circle Occupy

The Constant Radius $\rightarrow |M_{EAB}B| = \frac{1}{2} \sqrt{2 \cdot s^2 - |KB|^2} = \text{Constant} \dots (f-2) \text{ OR ,}$

The Locus of Any Point X of the Plane For Constant Sum-Squares from Constant Edge Points E_B , E_A , of Line - Sector , Is The Circle with Center the Mid-Point |M_{EAB}| of The Sector , And Radius The Diametus |XM_{EAB}| of Triangle |XE_B E_A| .

**C3.. THE PROOF FOR TWIN - COUPLE TRIANGLES METHOD FOR
THE CONSTRUCTION OF THE REGULAR - ODD – POLYGONS ,
→ USING A RULER & A COMPASS ←**

Data in [Fig- 6-4].

C3a..In the System of circle $[O, OK]$, of Diameters $KK' \perp aa' = 2R$, $KK' \perp K(E)$ and Chords $KA = KE_A$, $KB = KE_B$, Represent The Fixed Twin - Couple of Triangles KAK' , $KA E_A$, $KA E_B$. where KA, KB , Is an $n, n+2$, Even Sequence Set **The** Circle of Apothems $[K, KA_A]$ **Denotes** at Point A_A , **The** line Sector $|A_A K| = |A_A A|$ And By Extending The Foot of the Apothem to the Circle $[O, OK]$ at Point A_{1A} , is Defined *According to Archimedes* the $2n$ -Sided Polygon which is for Points A, B , The Octagon at Point A_{1A} , Dodecagon at Point A_{1B} , and **Decagon at Point A_{1C}** . **The** Circle $[K, KB = KB' - KO]$ of Constant Squares Difference from the Points E_A, E_B , **Denotes** at Points B, B' , M_{AB} the Constant Difference of Squares from Constant Points E_A, E_B . **The** Circle of The **Constant Sum** of Squares from the Midpoint M_{EAB} of Distance $|E_A - E_B|$ **Denotes** at Points B, B' , M_{AB} , The Constant Radius $|M_{EAB} M_{AB}|$ of the Circle with Center The Midpoint M_{EAB} . **ALL above Constructions of , Points , Circles , Line Sectors , Use a Ruler and a Compass .**

C3b..THE GEOMETRICAL - PROOF :

- a)..On the Isosceles Triangle $KB E_B$ issues **The Constant Squares Difference** $|BK|^2 - |BE_B|^2$, from Points K, E_B , **The Constant Sum of Squares** $|BK|^2 + |BE_B|^2$ from Midpoint O_{BE} of Distance $KE_B = KB$, and **The Angles** $\angle K \hat{B} E_B = \varphi_B = \angle B \hat{K} E_B = \angle K \hat{O} A_B$,, **Because Above Properties exist for Point B ,, Therefore ,, Point B exist On the Locus $B-B'$** .
- b)..On the Circle of Apothems $[D_{B1}, |D_{B1}K| = |D_{B1}O|]$ are **The Foots A_B** and when extended to the circle $[O, OK]$ as Points A_{1B} these are **The Vertices** of the $2n$ -Sided Regular Polygon. Because Apothems are Perpendicular to the Mid-Sides , Angles $\varphi_{KO A_B}$ at Center Point O are equal to φ_B , or $\varphi_B = \varphi_{KO A_B}$, Analogically The Vertices of the $2n$ -Sided Regular Polygons Belong to Even Polygons . **Therefore** By determining the Midpoint of the **Arc** $A_{1A} - A_{1B}$, Is thus Determined **Point A_{1C}** which is the Vertex of the $2n$ -Sided Inbetween Regular ODD Polygon . For $n = 4$ exists $\rightarrow$ Octagon at Point A_{1A} , $n = 6 \rightarrow$ Dodecagon at Point A_{1B} , and for $n = 5 \rightarrow$ **The Decagon at Point A_{1C}** with Foot A_C to be **Common** to the **Circle of Apothem** $[D_{B1}, |D_{B1}K| = |D_{B1}O|]$ and to the **Line Sector** OA_{1C} ,
- c).. By Extending the lines KA_A, KA_B, KA_C , with the Foots of Apothems A_A, A_B, A_C , to Points B', B, M_{AB} , of circle $[K, KB]$, **Define Point M_{AB} of the Regular Decagon** . Because Points on circle $[K, KB]$ occupy the Property of **Constant Sum of Squares** the Same issues for Point M_{AB} , **i.e. The Radius $|M_{EAB} M_{AB}|$ is Constant ,, Therefore ,,** By Drawing the circle from Midpoint M_{EAB} and Radius $|M_{EAB} M_{AB}|$, intersects circle $[O, OK]$ at Point F_{n+1} , **which is the Vertex of the $2.[n+1]$ Regular Polygon ...o.e.δ.**

C3c.. THE REGULAR TRIGONE :

Chord KK' is the Side λ_A of the Regular Digone in circle (O, OK) with Point D_{A1} at Point K and is the Constant Projection of Point , and Chord $KA = KE_A$ is the Side λ_B of the Regular Tetragon in circle (O, OK) . The Circle of **Apothems** is the Circle $[D_{B1}, D_{B1}K]$ on Diameter KO , and Archimedes **2n-Sided** Polygons is circle (O, OK) . The Locus of the **Constant-Difference** of Squares $[|BK|^2 - |BE_A|^2]$ is the Mid-Perpendicular on KO Chord , which denotes Point B of the circle (O, OK) . The Locus of the **Constant - Sum** of Squares $[|BK|^2 + |BE_A|^2]$ is The circle with Center the Mid-Point O_{AE} of KE_A and Radius $[O_{AE} B]$ which Intersects main circle at Point B . From above Pont B is the Vertex of the $2n=2 \times 3= 6$ In-Between Regular Hexagon , **Therefore** ,

Point **B** is the Vertex of Regular-Trigone and $|K'B|$ The Side of the Regular Trigone .

C3d.. THE REGULAR PENTAGON :

For the Regular Pentagon the central Angle $w_5 = [\frac{2}{5}].180 = 72^\circ$ and $\varphi_5 = 36^\circ$, therefore issues Angle $\widehat{EA_{k+2}K} = 72^\circ - 36^\circ = 36^\circ = \varphi_5$, and so Side $[EA_{k+2}] = EK$. Besides the

$\lambda_{2.N}$ Polygon in the same circle is $\lambda_{2.N} = \sqrt{2R^2 - R\sqrt{4R^2 - \lambda_N^2}} = [A_{k+1} K'] = [A_{k+2} K']$ and , $A_{k+2} D \parallel K'A_{k+1}$, and issues $\rightarrow [KE] = [EA_{k+2}] = [A_{k+1} A_{k+2}]$,

The Apothem $KM_{k+1} = 2.A'O$, The Sides $[KA_{k+1}] = [A'A_{k+2}] = 2.A'O$, **so that to**

Be Possible the Construction of the 2-n Polygon , where then equation (5) becomes

$[A_{k+1} A_{k+2}]^2 = [2.A'O]^2 - [A'A_{k+1}] * [A_{k+1} A_{k+2}] \dots\dots (6)$, and By Placing

$[A_{k+1} A_{k+2}] = x$, $[A'A_{k+2}] = 2.R \rightarrow x^2 = [2R]^2 - 2R.x \leftarrow x^2 + 2R.x - 4R^2 = 0 \dots(7)$

The second Degree Equation (7) occupies the following Roots .

$$x_{1,2} = -\frac{2R}{2} \pm \frac{1}{2} \sqrt{4R^2 + 16.R^2} = -R \pm R.\sqrt{5} = R.[\sqrt{5} - 1]$$

The Method Applied on Pentagon :

The 2-Even Polygon Set---- $[n, n+2]$ --- are the Sides KA of Regular -Tetragon and KB of Regular-Hexagon . Chord KA is the Side λ_A in circle (O, OK) and $KE_A = KA$ in circle (K, KA) while Point D_A is the Constant Projection of Point A . Chord $KB = KE_B$ is the λ_B Side in circle (O, OK) and $KE_B = KB$ in circle (K, KB) , while Point D_B is the Constant Projection of Point B . The Circle of **Apothems** is the Circle $[D_{B1}, D_{B1}K]$ of Diameter KO where The Chords are Divided , and **Archimedes 2n-Sided** Polygons is always On the circle (O, OK) . The Locus of the **Constant-Difference** of Squares $[|BK|^2 - |BE_B|^2]$ is the Circle $[K, KB]$. The Locus of the **Constant - SUM** of Squares $[|BK|^2 + |BE_B|^2]$ is the Circle from Mid-Point O_{BE} , and Radius is The Diamesus $|O_{BE}B|$.

By Extending The Foots A_A, A_B of the Apothem to the Circle $[O, OK]$ at Points A_{1A}, A_{1B} is then Defined *According to Archimedes* the $2n$ -Sided Polygon which is for Points A, B , The $2n$ -Octagon at Point A_{1A} and $2.[n+2]$ -Dodecagon at Point A_{1B} . **Since** $2.[n+1]$ Regular **Decagon** must Be , *According to Archimedes* , in the Mid-Pont A_{1C} of Circle $[O, OK]$,

Therefore , Line-Chord $|OA_{1C}|$ Interswcts The Circle of Apothems at Point A_C , which

Is The Foot of Apothem $|OA_C|$ and Thus is Defined A_C on Apothem Circle.

Extending Line-Sector $|KA_C|$ to Point M_{AB} , of the Circle $[K, KB]$ of Constant-Difference Squares, Point M_{AB} is on the Locus of the **Constant-Sum** Squares from Mid-Point M_{EAB} , **Therefore, The Circle of Constant-Sum-Squares ($M_{EAB}, M_{EAB} M_{AB}$) Intersects Circle $[O, OK]$, at Point F_5 , which is the Vertex of the Regular $2.[n+1] = 2 \times 5 = 10$ & Chord $|KF_5| = |KC|$, The SIDE of the REGULAR – PENTAGON ...o.ε.δ.**
OR
The Outer are The Two Even - Regular n & $n+2$ Polygons ,
And The Inner is $\rightarrow$ The $n+1$ ODD – Regular Polygon $\leftarrow$

Remark :

Since Point **B** is on The Circle of **Apothems** and $|BK| = |BO|$, the line-Sector $|D_{B1}B|$ is The Locus of **Constant-Difference** Squares $[|BK|^2 - |BE_A|^2]$, And The Locus of the **Constant -Sum** Squares . When line-Sector $|D_{B1}B|$ is Produced , Intersects $|AD_A|$ at Point A_1 where issues $|AA_1| = h_A - h_B = \Delta h$, **Therefore**

The Harmonic Mean issues as $\sin \varphi_a \cdot \sin \varphi_b = \sin^2 \varphi_o$, where $\sin \varphi = \left(\frac{h_\varphi}{\lambda_\varphi}\right)$ on Difference of Heights $[h_A - h_B]$, Between the Apothems $[r_a - r_b]$,

C3e.. THE REGULAR HEPTAGON :

The Method :

The 2-Even Polygon Set--- $[n, n+2]$ **— are the Sides KA of Regular -Hexagon and KB of Regular - Octagon . Chord KA is the Side λ_A in circle (O , OK) and $KE_A = KA$ in circle (K , KA) while Point D_A is the Constant Projection of Point A . Chord KB = KE_B is the λ_B Side in circle (O , OK) and $KE_B = KB$ in circle (K , KB), while Point D_B is the Constant Projection of Point B . The Circle of **Apothems** is the Circle $[D_{B1}, D_{B1}K]$ of Diameter KO where $[D_{B1}K] = [D_{B1}O]$, and Chords are Divided , and **Archimedes 2n-Sided Polygons** is always On the circle (O , OK) . The Locus of The **Constant - Difference** of Squares $[|BK|^2 - |BE_B|^2]$ is the Circle $[K, KB]$. The Locus of the **Constant - SUM** of Squares $[|BK|^2 + |BE_B|^2]$ is the Circle from Mid-Point O_{BE} , and Radius the Diametus $|O_{BE}B|$.**

By Extending The Foots A_A, A_B of the Apothem to the Circle $[O, OK]$ at Points A_{1A}, A_{1B} is then Defined *According to Archimedes* the 2n-Sided Polygon which is for Points A , B , The 2n- Dodecagon at Point A_{1A} and 2. $[n+2]$ -Decaexagon at Point A_{1B} . **Since $2.[n+1]$ Regular Decatetragon** must Be , *According to Archimedes* , in the Mid-Pont A_{1C} of Circle $[O, OK]$ **Therefore , Line-Chord $|OA_{1C}|$ Intersects The Circle of Apothems at Point A_C , which**

Is The Foot of Apothem $|OA_C|$ and Thus is Defined A_C on Apothem Circle.

Extending Line-Sector $|KA_C|$ to Point M_{AB} , of the Circle $[K, KB]$ of Constant-Difference Squares, Point M_{AB} is on the Locus of the **Constant-Sum** Squares from Mid-Point M_{EAB} ,

Therefore , The Circle of Constant-Sum-Squares $[M_{EAB}, M_{EAB} M_{AB}]$ Intersects Circle $[O, OK]$, at Point F_7 , which is the Vertex of the Regular $2.[n+1] = 2 \times 7 = 14$ & Chord $|KF_7| = |KC|$, The SIDE of the REGULAR – HEPTAGON...o.ε.δ.

THE GEOMETRICAL CONSTRUCTION OF THE **ODD**
REGULAR - HEPTAGON $n+1 = 7$, FROM THE
TWO EVEN **REGULAR** $n = 6$ **HEXAGON** AND
THE EVEN **REGULAR** $n+2 = 8$ **OCTAGON**

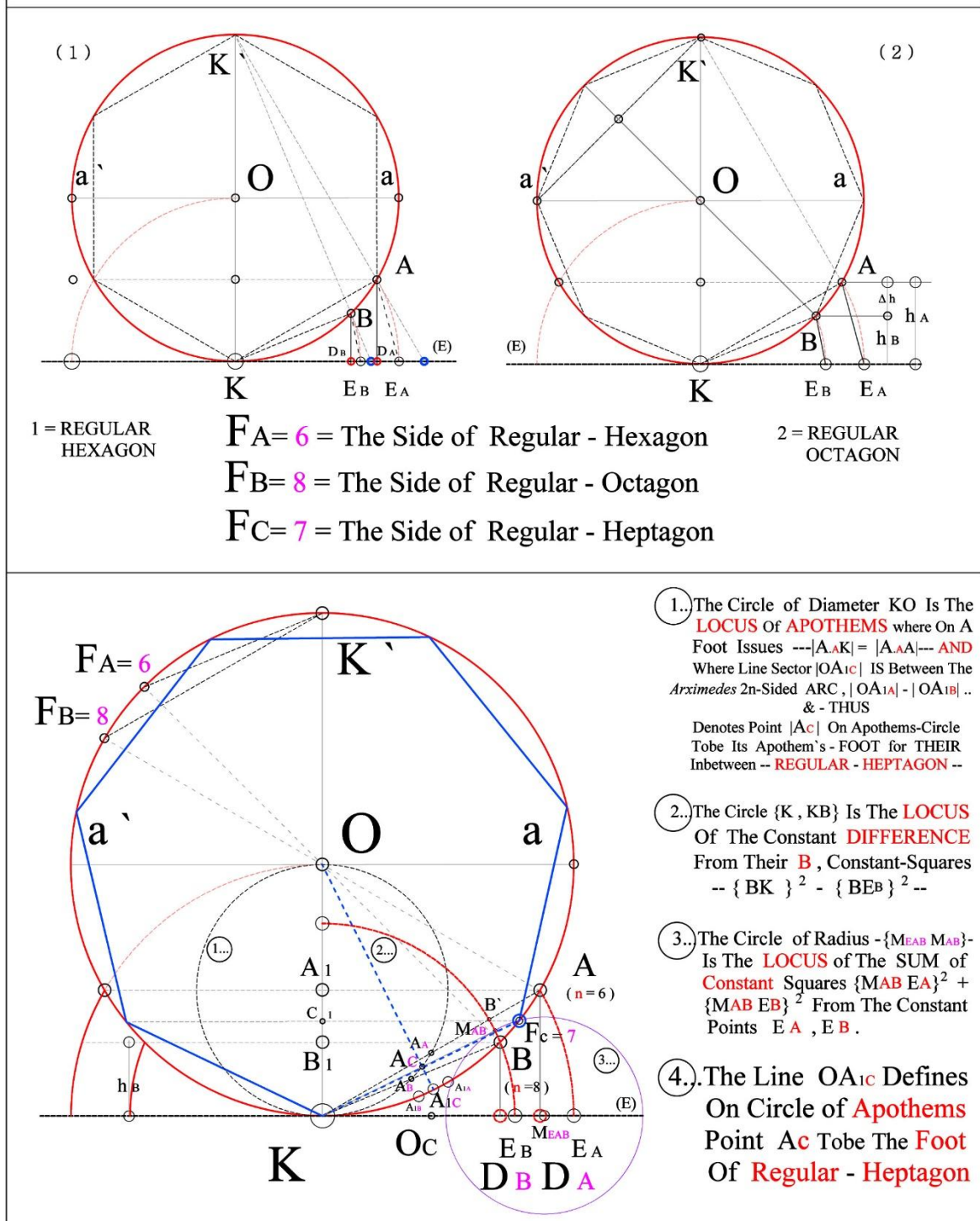


Figure-7- The Regular Heptagon :

C4.. THE Geometrical Method of Constructing ALL Regular Polygons : & Decaheptagon

The 2-Even Polygon Sets---[$n, n+2$]— are the Sides KA of n =Regular Deca-Hexagon and KB of The $n+2$ =Regular Deca-Octagon . Chord KA is the Side λ_A in circle (O , OK) and $KE_A = KA$ in a circle (K , KA) while Point D_A is the Constant Projection of Point A . Chord KB = KE_B is the λ_B Side in circle (O , OK) and $KE_B = KB$ in circle (K , KB) , while Point D_B is the Constant Projection of Point B . The Circle of **Apothems** is the Constant Circle [$D_{B1}, D_{B1}K$] of Diameter KO where The Chords are Divided , $|D_{B1}O| = |D_{B1}K|$ and Archimedes **2n-Sided** Polygons is always On the circle (O , OK) . The Locus of the **Constant-Difference** of Squares [$|BK|^2 - |BE_B|^2$] is the Circle [K , KB] . The Locus of the **Constant - SUM** of Squares [$|BK|^2 + |BE_B|^2$] is the Circle from Mid-Point O_{BE} , and Radius the Diametus $|O_{BE}B|$.By Extending The Foots A_A, A_B of the Apothem to the Circle [O,OK] at Points A_{1A}, A_{1B} is then Defined , *The Archimedes 2n-Sided Polygon* which is for Points A , Point A_{1A} and for The 2.[$n+2$]- at Point A_{1B} . **Since** 2.[$n+1$] Regular Polygon must Be , *According to Archimedes* , in the Mid-Pont A_{1C} of Circle [O,OK] ,,,**Therefore** ,, **Line-Chord $|OA_{1C}|$ Intersects The Circle of Apothems at Point A_C** , which **Is The Foot of Apothem $|OA_C|$ and Thus is Defined A_C on Apothem Circle , and for the Set The Mid-Pont A_{1C} Belongs to Triantatetragon , and The Foot A_C to the Decaheptagon .** Extending Line-Sector $|KA_C|$ to Point M_{AB} , of the Circle [K , KB] of Constant-Difference Squares , Point M_{AB} is on the Locus of the **Constant-Sum** Squares from Mid-Point M_{EAB} , **Therefore. The Circle of Constant-Sum-Squares [$M_{EAB}, M_{EAB} M_{AB}$] Intersects Circle [O,OK], at Point F_{17} , which is the Vertex of the Regular 2.[$n+1$]= $2 \times 17 = 34$ And The Chord $|KF_{17}| = |KC|$, Is The SIDE of the REGULAR DECAHEPTAGON. In circle (O , OK) ..o.e.δ.**

Remarks :

- 1... The Method Solves the Problem of The Regular Polygons in Plane Using a **Circle with Two Perpendicular axis KK' , aa'** , and An Even-Sequence-Set of Chords. This Process allows The Rotation of The System through the Two axis separately and Thus is Possible **The Spatial Construction of the Regular Polygons** . In a Sphere with Infinite great circles within The Sphere is Possible By Rotation on and Through the Two Perpendicular axis , to Design all Sequences of The Even Regular - Polygons with their Inbetween Odd Polygon into The Spatial Geometry .
- 2... The Precise Definition of Points on lines and Circles enables the Programming of The Method to Use The Coordinate Systems (x, y, z) , r, ϕ , (i, j, k) For Algebrical Solutions and Others .
- 3... Because The Spaces—Anti-Spaces Exist simultaneously on their Great Circles of The Spheres , Then All Informations can Immediately Transferred to Each - Other .

D... THE TWIN – COUPLE TRIANGLES Method FOR CONSTRUCTING THE REGULAR --- Odd --- POLYGONS . IN SPATIAL -- GEOMETRY .

1d... The Spatial Regular Polygons .

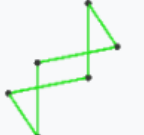
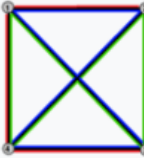
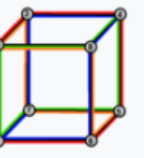
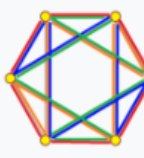
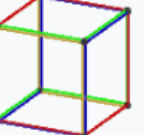
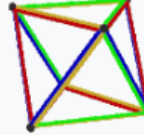
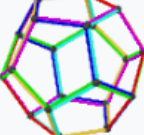
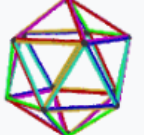
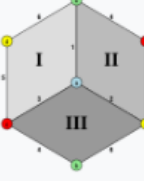
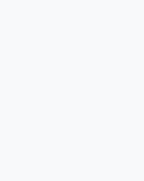
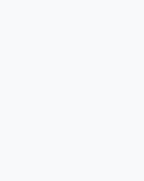
Regular petrials					
Name	Petrial tetrahedron	Petrial cube	Petrial octahedron	Petrial dodecahedron	Petrial icosahedron
Symbol	$\{3,3\}^\pi$	$\{4,3\}^\pi$	$\{3,4\}^\pi$	$\{5,3\}^\pi$	$\{3,5\}^\pi$
$(v,e,f), \chi$	$(4,6,3), \chi = 1$	$(8,12,4), \chi = 0$	$(6,12,4), \chi = -2$	$(20,30,6), \chi = -4$	$(12,30,6), \chi = -12$
Faces	3 skew squares 	4 skew hexagons 		6 skew decagons 	
Image					
Animation					
Related figures	 $\{4,3\}_3 = \{4,3\}/2 = \{4,3\}_{(2,0)}$	 $\{6,3\}_3 = \{6,3\}_{(2,0)}$	 $\{6,4\}_3 = \{6,4\}_{(4,0)}$	 $\{10,3\}_5$	 $\{10,5\}_3$

Figure-8

In Chemistry , Atoms follow the Spatial Geometry where .

In an Octava of Electrons Protons are Nested in the **In-Sphere** $\equiv n=1-122$ Regular Triangles , The In-Sphere is Nested In the **Regular Tetrahedron** , which is Dual with the 3-Diagonals of The Cube , And the Cube is Nested in The **Ex-Sphere** which are the Orbits of Electrons .

2d... The Geometrical & Physical Notion of the Spatial Regular Polygons :

The 4-Geometrical Spaces $\rightarrow$ In-Sphere , Tetrahedron , Cube , Ex-Sphere $\leftarrow$ consist the Basic Geometrical Moulds . relation for Plane Work , which is The Quantization of Geometry Shapes into the Plane – Stores [The Surfaces] of Anti-Space , and consist The Unification of Geometry–monads with those of Energy monads , and which were analyzed and have been fully described , by markos 20/8/2017

The **Vectors of Amplitude** ON the 4 -Spaces consist the **E-Form-Mould** . and is constructed **in Space** as $\rightarrow$ The Inscribed and the Circumscribed Sphere as **Regular Tetra Pleuron** , and **in Anti-Space** $\rightarrow$ The Inscribed and the Circumscribed to the Regular **Tetra Hedron**, **Regular Penta Hedron** $\leftarrow$ **in Sub-Space** as The Inscribed and the Circumscribed to the circle **Regular Hexa Hedron** $\leftarrow$ Continusley , Space Regular Polygons Into the Anti-space Regular Polygons.

3d... The Spatial Regular Polygons in Atoms Structure .

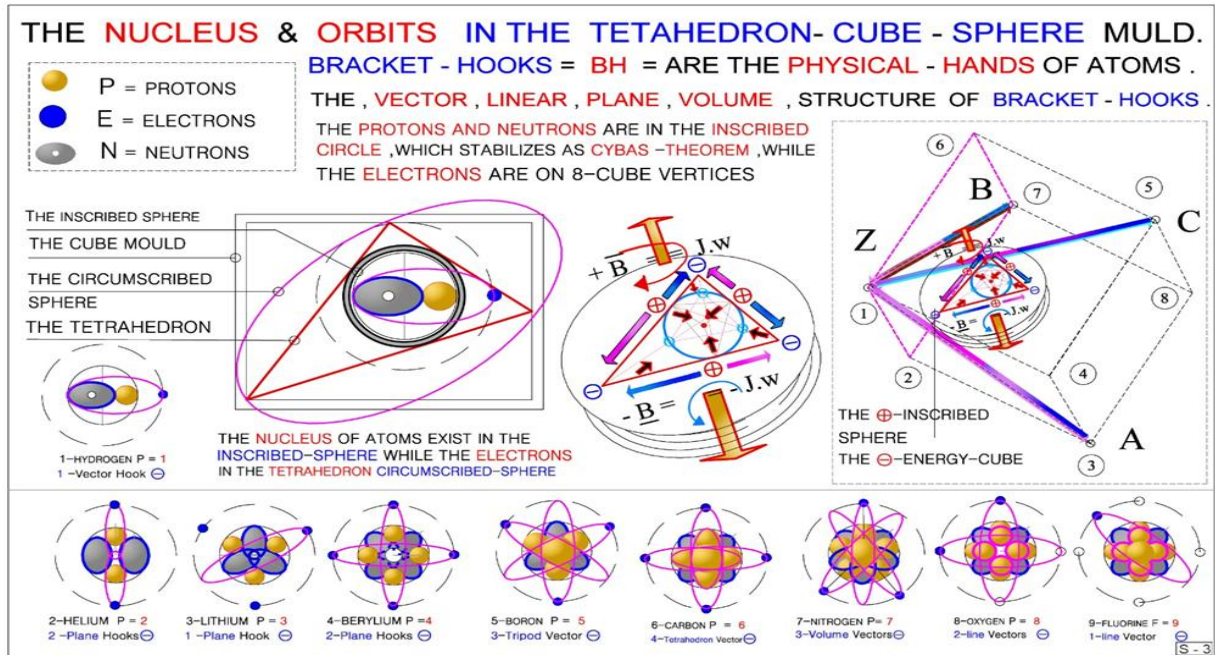


Figure - 9- : The Active-Conductors of Elements and The Space , Anti-Space formation .

In Figure **ZA , ZB , ZC** , are the three **Conductors** On the Three Diagonals of Cube with 1 , 2 , 3 , 4 , 5 , 6 , 7 , 8 , Vertices . At Point **Z** is Acting a **Complex-Vector $\bar{Z}$** and which transform the Vibration to the three conductors in **Space Pattern** . It was shown [61] that The Stresses of Complex-Vectors are Spread in Surfaces Only for Still states.

The number of Moulds of Stationary-States are $\rightarrow 2^{n=1-4 \text{ Points}} \equiv 2, 4, 8, 16 \equiv 4\text{-Types} \leftarrow$
 $[2] = \text{For the Proton} = \text{Nucleus}$, $[4] = \text{Tetrahedron}$, $[8] = \text{Cube} = \text{The Neutral-caves}$, $[16] =$
 The Double Number of Vertices for the 8-Stationary Neutral-caves & called **TEES-Pattern**

$\rightarrow \{ \text{The Sphere} - \text{IN TETRAHEDRON} - \text{IN Cube} , \text{IN External Sphere} \} \leftarrow$

The Active-Conductors of Elements and The Space , Anti-Space formation :

Point , in E-geometry is considered nothing , without Position and direction while in , **MG $\equiv$ Material - Geometry** [1 Positive $\oplus$ and 1 Negative $\ominus$] . From Nothing (i.e. the Point **Z**) to Existence (i.e. to Be another Spherical Point **P**) issues the **Zero Virtual work law** , where zero Work is the equilibrium of Two Equal and Opposite forces on Points and is $[\oplus - d - \ominus]$. Thus Space **[S]** is **Z-Point $\oplus$** , Anti-space **P-Point $\ominus$** , and **d = ZP = The Conductor of Points** . Since , **The Position and the Number of Points** create the Spaces so the Only Three elements are $\rightarrow \{ \oplus , [\oplus \leftrightarrow \ominus] , \ominus \} \equiv [+ , 0 , -] \leftarrow$ **for Material Geometry exist linearly in moulds .**

The Stresses of Complex-Vectors **Act** on a Surface , **A circle of Sphere** , between the Sub Units **Axially** and **Rotationally** following **Ptolemy's and Cebas Theorem** of Equilibrium.
 The Space Anti-Space and , Neutral-Space Conductors are self-Sustaining .

Remarks : The Work of Elastic Deformation .

Elastic Deformation requires (1) That every New Stress that is applied to a New infinite Element is always added to the existing one .(2).. Be always Unity Work of Deformation .

When a Body contains Potential Energy ,

During any Unloading it gives off Mechanical - Work , Since the Points of Application of the GRADUALLY - withdrawn Loads are Displaced.

If the Unloading is ABRUPT , Then Kinetic Energy is converted into Potential and after is Kinetic , converted Back into Kinetic and so on . **That is** , the Body Performs an Oscillating motion until the Mechanical Energy is Extinguished due to a State of Equilibrium or Alters .

4d... The Stability of the Cosmic Particles Conductors and Atoms – Nucleus . [109-111]

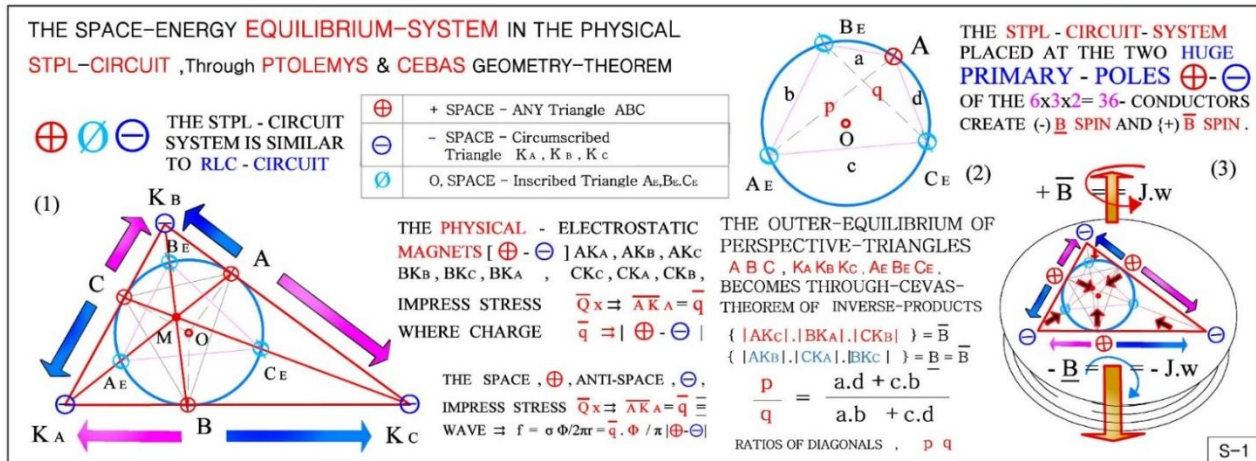


Figure -10- : The Stability of $\rightarrow \{ \oplus - \text{Space } [A B C] \}, \{ \ominus - \text{Anti-Space } [K_A K_B K_C] \}$, of The Sub-Space $[A_E, B_E, C_E]$, and Neutral-Space $\{ [\oplus \leftrightarrow \ominus] \equiv ds \equiv [\oplus \leftarrow ds \rightarrow \ominus] \}$. SINCE , *Energy of The first Wheel-Rim or Orbital is distributed to the Couple of (2e) Electrons of the Orbital , And Then this is a Cycloid - Anti-Cycloid - Stability System .* This Still Systems of Space –Anti space Regular Spatial Polygons **DOES NOT mean Matter–Antimatter But The State of Equilibrium of Space and Anti Space . $\rightarrow$ The State of Equilibrium in Caves $\equiv$ The Ceba's Energy triangle . $\leftarrow$**

Remark :

The Stability of Spaces–Anti-spaces Exists from the Simultaneous Actions on Their Regular Sets of Polygons , And Informations can Immediately Transferred to Each - Other through Their Common Spatial Path which is the Common Sphere .[100]

The Conductors occupy In-Plane the Triangle which is the most Steady Figure , and in Space The Regular Tetrahedron which is the first Spatial Figure .

5d... THE TETRAHEDRON IN CUBE IN EX-SPHERE MECHANISM. [109-111]

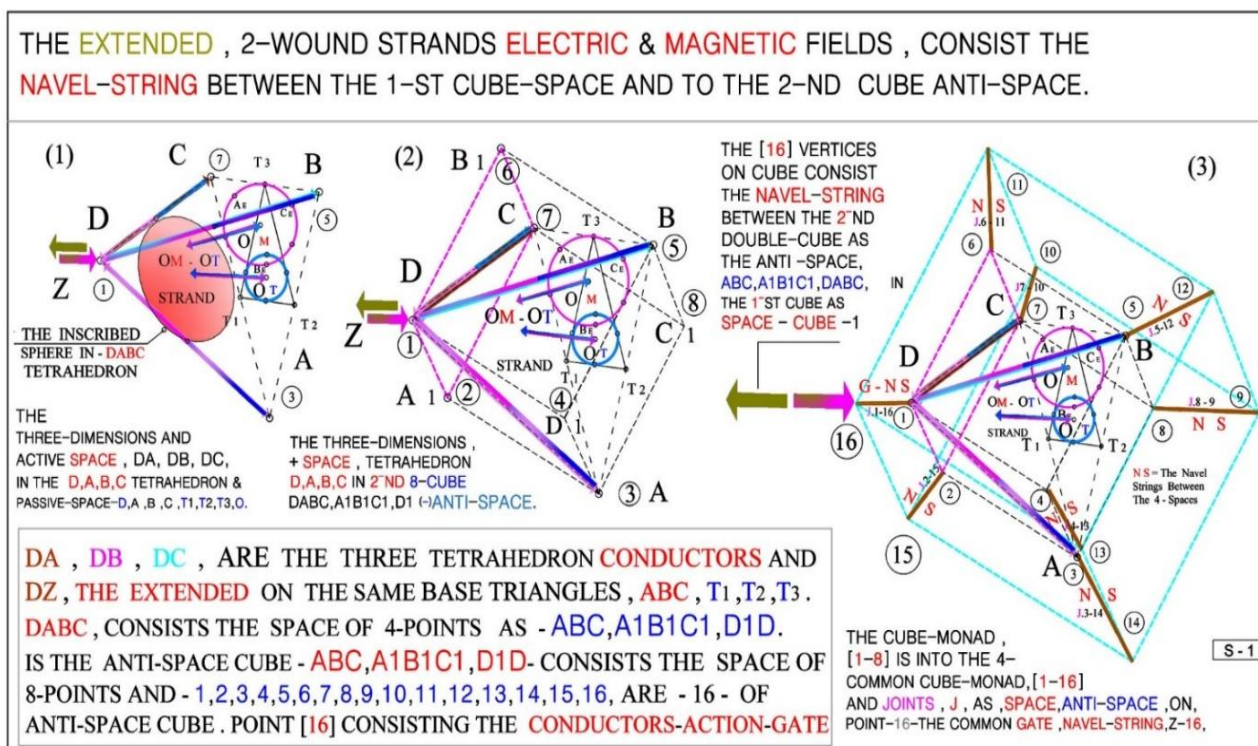


Figure – 11- : The Electric - Magnetic Field of 3 - Conductors Results on 2 -Anti-Parallel - Strands .The Stability of $\rightarrow \{ \oplus 4\text{-Space } [D A B C] \}$ Regular - *Tetrahedron* , IN $\{ \ominus 8\text{-Anti-Space Cube } [D A_1 B D_1, C C_1 A B_1] \}$, and The $\rightarrow \{ \oplus 8\text{-Space } [D A_1 B D_1, C C_1 A B_1] \}$ *Cube*, INTO The $\{ \ominus 16\text{-Anti-Space } [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16] \}$ *Sphere Cube Vertices* . The Point [16 $\equiv | \Rightarrow |Z|$] consists the **Navel - String Gate in the System .**

and are $\{ Z - (\Delta D, ABC), \Delta [T_1 T_2 T_3] \}$ Stationary Bases . The Stability Analysis in [88]

6d... The Matter Antimatter in 3-D Origination Mechanism $[\oplus \leftrightarrow \ominus]$ [103]

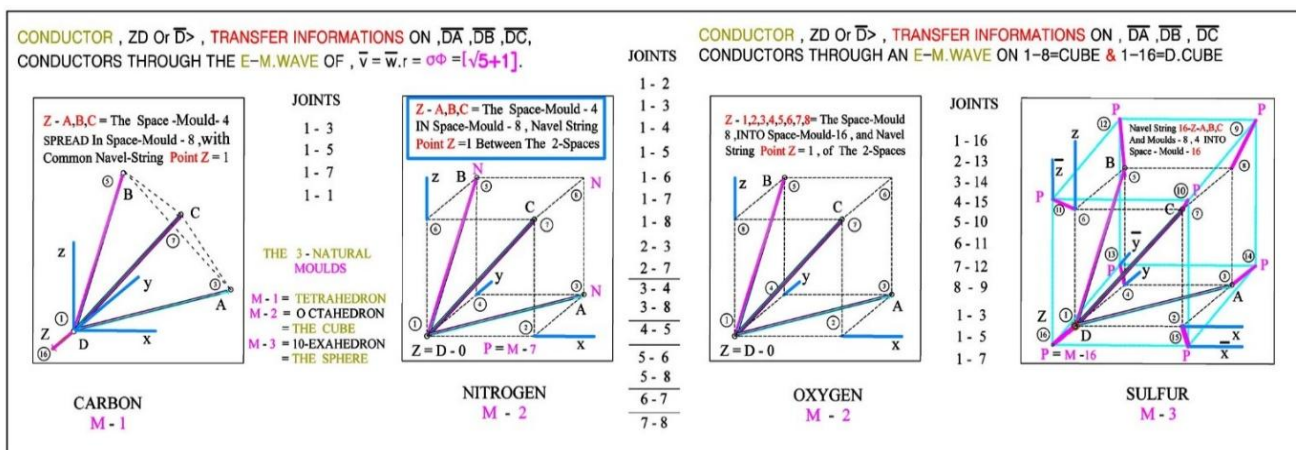


Figure – 12 - : The 4-Dimensional Spinning , Tetrahedron - Cube – Spheres - Mould .

The **Stationary-States** are $\rightarrow 2^{n=1-4} \text{ Points} \equiv 2, 4, 8, 16 \equiv 4\text{-Types} \leftarrow$

7d... The In-SPHERE In-TETRAHEDRON In-CUBE In-EX.SPHERE MOULD .[103]

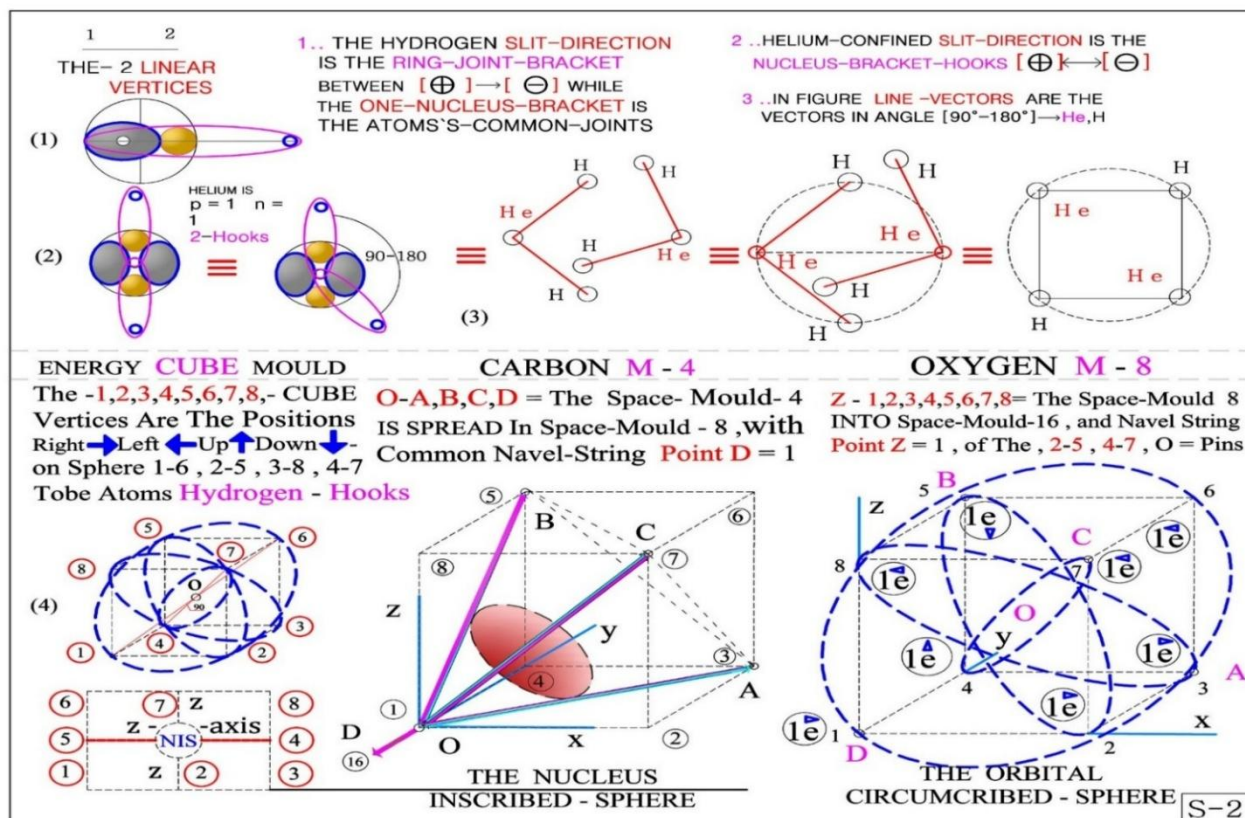


Figure – 13.. :The Hydrogen , Carbon , Oxygen , Atoms-Conductors Into the Sphere-Cube-Mould . Deka Hexahedron , Octahedron $\equiv$ Cube , Tetrahedron $\equiv$ 4-Points , Triangle .

The Nucleus $\equiv$ (In-Sphere) , Tetrahedron $\equiv$ (D - A.B.C) ,

Cube $\equiv$ (O,2,A,4,B,6,C,8) , Orbitals $\equiv$ (Ex-Sphere) , Mould .

When **Vector $\equiv$ 2-Points $\equiv$ The Closed-Conductor** , 1-Point $\equiv$ The Opened -Conductor $\equiv$ The Bracket-Hook . Oxygen is the Element which fills 8 - 2 = 6 vertices of Cube as EL $\rightarrow$

$$\begin{matrix} 1e & 0e & 1e \\ \{ 1e & \vec{2} & 0e \} \equiv (\pm \frac{2}{2}) , \text{ where } \textcircled{0} \text{ is the first , Zero-Acting Cube filled with } 1e , \\ 1e & 1e & 1e \end{matrix}$$

Therefore is the only Steady Element which occupies the first Energy-Level $\textcircled{1}$

with 2 Electron-Positions as $\textcircled{1} = 2e = \vec{2}$ Positions at $\vec{1}, \vec{1}$

The Electron Being in the Hydrogen Precesses Because of the different axis of Rotation and Nutation's , from the continuous and immense-communication to the Effect of gravity **g** .

Since Electron is continually oscillating , with the Nutation-frequency **f_N** , so Produces a continuous and Oscillating Magnetic-field –M- which in turn is the source of an oscillating Electric-field – E - which implies the **Regeneration of each other** , i.e. it is a **Propagating Electromagnetic-Wave** where **E = B c** , and with a Quantum-Energy **E = h f_N** or **E = 2μ.B** , This Energy E , consists the Hydrogen Bracket **Hook [HBH]** , or **[BH]** and it is the only **free monad** which can **Bond** to all Energy Structures .

The **Spheres - Cube-Structure of Atoms** In Sphere Center –O- allows **The Stability** of the System , Per–Two electrons in Dipole Positions , $1 \leftrightarrow 6$, $2 \leftrightarrow 5$, $3 \leftrightarrow 8$, $4 \leftrightarrow 7$, consisting The-Positive Dipole- $[\overrightarrow{BH}]$, to Be able to move $\rightarrow$ Into-In- Outside it $\leftarrow$ and allow the Atoms-Bonding as this in Energy levels .*The Cube-Structure of Atoms and Orbits in the Out-Sphere* formulates + $[\overrightarrow{BH}]$ to those **Dipole Positions which lack of one electron** and so can **PLUG -In** the **LOOP** of the others . The Atoms with Zero Stable State are ,

The Hydrogen Bracket-Hook **Into the Carbon** encloses the Inscribed Sphere for the Nucleus , **Into the Oxygen** for the Orbitals , **Into the Sulfur** enclosed into the Circumscribed Sphere for the Atoms-Conductors Sustains .

The First 4-Dimentional Spinning ,**Tetrahedron-Cube-Sphere-Mould** are , $9c-2$

- 1... **O-1-3-5-7 -** $\rightarrow$ **The First Solid Tetrahedron Structure in Nature** ,
- 2... **O-1-2-3-4-5-6-7-8** $\rightarrow$ **The First Cube with max. Positions $e = 2.n^2 = 8$** ,
- 3... **R - [O , A , B , C]** $\rightarrow$ **NUCLEUS , IS IN the First Inscribed Sphere into Tetrahedron.**
- 4... **R-O-1, 2, 3, 4, 5, 6, 7, 8** $\rightarrow$ **ORBITS ARE IN Enveloped Cube-Sphere of Tetrahedron.**

The Deka Hexahedron , Octahedron $\equiv$ Cubes , Tetrahedron $\equiv$ 4-Points , Triangle $\equiv$ 3-Points , Vector $\equiv$ 2-Points $\equiv$ The Closed-Conductor , 1-Point $\equiv$ The Opened -Conductor $\equiv$ The Bracket-Hook .

The Hydrogen **Nucleus** is kept in the Inscribed to the **Tetrahedron-Cube-System** where exist the Protons and Neutrons , while The Electrons of Hydrogen **Orbits** are kept into the Circumscribed, to the **Tetrahedron – Cube - System** . Both Equilibrium by following the Menelaus - Ceba's Principles in The Regular Triangles , Hexagons , Octagons . Decaexagones ... [70]

Any Two Perpendicular **Energy-Vectors** , $|\mathbf{Q} \Rightarrow \overrightarrow{AK_A}| = \leftrightarrow |$, $|\mathbf{Q} \Uparrow \overrightarrow{AK_A}| = \Uparrow |$ of this

Propagating Energy , FORM The **Energy-Square** , $|\leftrightarrow| \cdot |\Uparrow| = |\mathbf{Q} \Rightarrow | \cdot |\mathbf{Q} \Uparrow| \equiv q_0$, on Conductor AK_A and **Then** Energy-Circular-Prism = $|AK_A| \cdot \{ \mathbf{Q} \Rightarrow \overrightarrow{AK_A} \} \cdot \{ \mathbf{Q} \Uparrow \overrightarrow{AK_A} \} \equiv [\overline{\mathbf{Q}}_x \overline{\mathbf{Q}}_y] \cdot |AK_A| = |\leftrightarrow| \cdot |\Uparrow| \cdot |AK_A|$, i.e.

An Energy-Circular-Cone which is a Moving Energy -Volume , and **Because** from Cauchy equations of Stresses are of three Dimensions , **Then The Energy Stresses** remains Flat ONLY- WHEN the Plane-Section is a circle $[E_2]$ and

The Square becomes circle as The , **Anti-Quadrature..**

Because from Mechanics , the Hydrogen **Nucleus** is kept in the Inscribed to the **Tetrahedron-Cube-System** where exist the Protons and Neutrons , while The Electrons of Hydrogen **Orbits** are kept into the Circumscribed , to the **Tetrahedron–Cube-System** so . Both Equilibrium by following the Menelaus - Cebas Principles .

Oxygen is the Element which fills $8 - 2 = 6$ vertices of Cube and for Neon $10 - 2 = 8$, as

$$\begin{array}{ccc} 1e & 0| & 0/ \\ \mathbf{Ox} \rightarrow \{ 1e & \overrightarrow{2}| & 1e \} \equiv \left(\begin{smallmatrix} +2 \\ -2 \end{smallmatrix} \right) , \quad \textcircled{2} \equiv \mathbf{Ne} \overrightarrow{2}_{1x8} = \overrightarrow{2} + 8 = 10 \rightarrow \begin{array}{ccc} 1 & 1 & 1 \\ 1 & \overrightarrow{2} & 1 \\ 1 & 1 & 1 \end{array} \end{array}$$

The Possible Orbits of Oxygen are Two as , $\rightarrow 1e /- \overrightarrow{2} /- / 0 \leftarrow$ and $\rightarrow 1e | - \overrightarrow{2} | - 0 | \leftarrow$ for all the 8-Cube Positions . $\textcircled{0}$ is the first , **Zero-Acting Cube** and is filled with 1e , therefore is the Only Steady Element which occupies the first Energy-Level $\textcircled{1}$ with 2 Electron-Positions as $\textcircled{1} = 2e$ Positions at $\overrightarrow{1}$, $\overleftarrow{1}$. In the same way $\textcircled{1}$ is the first **One-Acting Cube** and is filled with 2e, and so on , The Electron being in the Hydrogen -cave Precesses because of the different

axis of Rotation and Nutation's , from the continuous and immense-communication to the gravity g . Since Electron is continually oscillating with the Nutation-frequency f_N so it Produces a continuous and oscillating Magnetic-field $-M-$ which in turn is the source of an oscillating Electric - field $-E-$ which implies the **Regeneration of each other** , i.e. it is a **Propagating Electromagnetic-Wave** where $E = B c$ and with a quantum-Energy $E = h f_N$ or $E = 2\mu.B$, This Energy E consists the Hydrogen Bracket **Hook HBH** , or $[\overline{BH}]$ and it is the only **free monad** which can **Bond** to all Energy Structures .

The **Sphere-Cube-Tetrahedron Structure of Atoms** In Sphere Center –O- allows to issue The **Ceba's Theorem for 6-Stresses** on the Inscribed Tetrahedron which Secure **The Stability** for the Nucleus-System , and for the Outer Electron-System the Stability is Obtained Per–Two electrons the Dipole Positions $1 \leftrightarrow 6$, $2 \leftrightarrow 5$, $3 \leftrightarrow 8$, $4 \leftrightarrow 7$, consisting The-Positive Dipole- $[\overline{BH}]$, to be able to move $\rightarrow$ Into-In- Outside it $\leftarrow$ and allow the Atoms-Bonding as this is in Energy levels .

The Cube-Structure of Atoms and Orbits in the Out-Sphere formulates the $+[\overline{BH}]$ to those Dipole Positions which lack of one electron and so can **PLUG-In** the **LOOP** of the others.

Conclusion :

- 1... The Forces of Atom's NUCLEUS , are Holded from $\rightarrow$ The Inscribed Sphere In The Tetrahedron , which follows the Ptolemy's and Ceba's Vectors Structure .
- 2... The Forces of Atom's ORBITALS , are Holded from $\rightarrow$ The Into the Cube , Circumscribed–Sphere Complex Vectors Structure .
- 3... The Forces of Atom's POSITIONS , are Holded from $\rightarrow$ The on Cube Inscribed Tetrahedron Complex Vectors Structure .
- 4... The Stability of the {TCIC} Mechanism follows the Ptolemy's & Ceba's Complex Vectors Structure [62]

The Action of the Complex–Vectors create the Waves of Motion $\equiv$ **Energy Waves** which are closed curves and Transported . The **Pattern** of Disturbances with the Informations Propagates from the One-Edge-Point to the other Edge-Point of Conductors , **OR** is on the [Sub-Units] .

Coulomb force $[\oplus \rightarrow \ominus] \equiv F_{[+ \leftrightarrow -]} = \frac{[\oplus \leftrightarrow \ominus]}{r^2} = \frac{[2\sigma]}{r^2} = F = \sigma . A = \left[\frac{2\pi r f}{\Phi} \right] . A = w r . \left[\frac{A}{\Phi} \right] = v . \left[\frac{A}{\Phi} \right]$, and Acts on a Surface , A , between the Sub-Units **Axially** and **Rotationally** following Ptolemy's and Cebas Theorem of Equilibrium .

Remarks : *The Work of Elastic Deformation On the { TCIS } Mechanism :*

Every New Stress that is Applied to D - Element is always added to the existing And the Unity Work of Deformation is transformed to three Tetrahedron Sides.

When a SIGNAL Strikes the TCIS – Cube's Mechanism .

The Signal is **Energy-Wave carrying motion** . Unloading Happens on the three **Still-Spaces in Tetrahedron - Sides** which withdrawn Loads are Displaced On the Anti-Parallel-Strand . If the Unloading is ABRUPT, then Kinetic Energy of 1-Strand is converted in the 2-Strand as Potential and so on . That is , CUBES Perform an Oscillating motion until the Mechanical Energy is extinguished to their Vertices , which are the stores of Information .

Anti-Parallel-Strand is the **Body containing the Potential Energy** , During any Unloading on CUBES , it gives off **Mechanical - Work** , since the Points of application of the Gradually - withdrawn Loads are Displaced . The Stability of Kinetic – Potential Energy is **continuously** .

Potential . From equation $F = \sigma . A = \left[\frac{2\pi r f}{\Phi} \right] . A = w r . \left[\frac{A}{\Phi} \right] = v . \left[\frac{A}{\Phi} \right]$ is seen that any loaded

Conductor of Area A , is related to the **Golden Ratio Pattern** Φ , either in **Plane or Vector**.

8d... The Figure-eight Knot & The Figure $n = 1 \approx \infty$, Knots Polygons : [114]

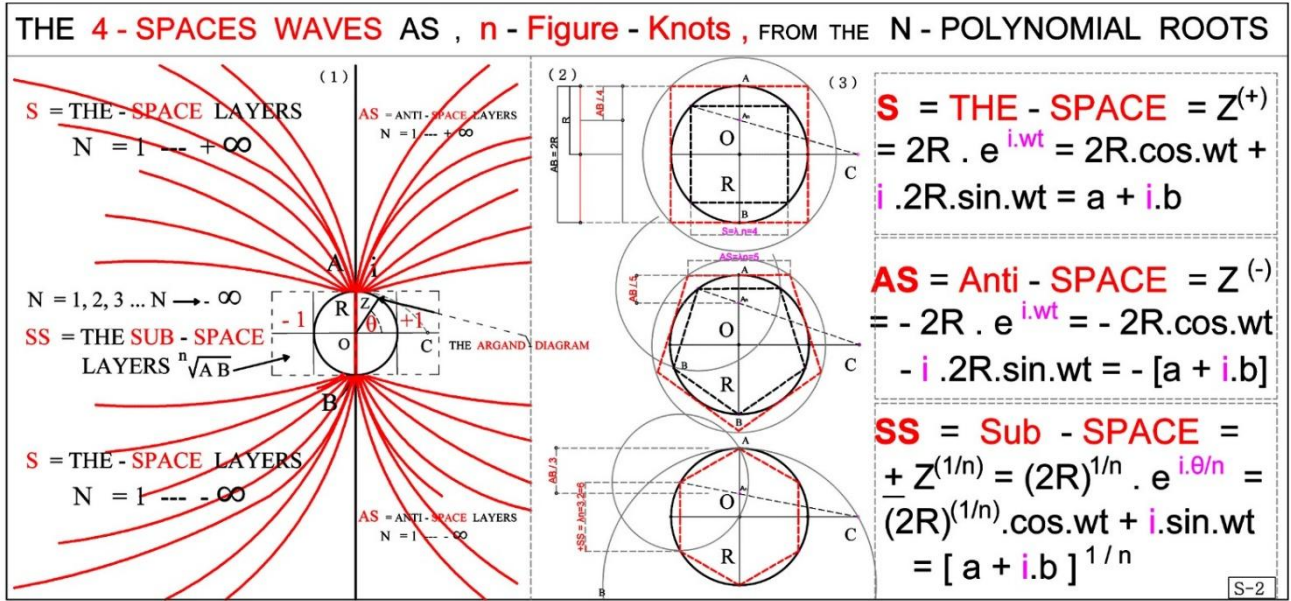


Figure - 14- : The Geometrical Construction of the 4-Spaces & the Spaces Waves of PNS .

In Argand Diagram $AB = 2R \rightarrow x = -1, O, +1, y = i$, exist the **Vector of Amplitude** $\bar{z} \equiv x + i \cdot y$, with angle $\theta^\circ = [Z - OC]^\circ$, $[i = \sqrt{-1}] \cdot x$. The **Wave's " strength "** and **" direction "**, where x is a number that represents the magnitude or length of the vector z , and from Definition of the **Complex Vector Space**, is the **Set of Vectors of length from The n Vertices** which form the n Polygon Sides, or the Norm of the Complex-Vectors .

In (1), (2), are the 4-Geometrical Spaces consisting The **Geometrical - Mould** .

In (2), are the **Vectors of Amplitude ON** the 4 -Spaces consisting the **E-Form-Mould** .

It is constructed **in Space** $\rightarrow$ The Inscribed and the Circumscribed to the circle of diameter AB , the $\rightarrow$ **Regular Tetra Pleuron** $\leftarrow$

It is constructed **in Anti-Space** $\rightarrow$ The Inscribed and the Circumscribed to the circle of Diameter AB , The $\rightarrow$ **Regular Pentagon** $\leftarrow$

It is constructed **in Sub-Space** $\rightarrow$ The Inscribed and the Circumscribed to the circle of Diameter AB , the $\rightarrow$ **Regular Hexagon** $\leftarrow$

In (3), are the **Norm (magnitudes)** of The **4 -Vectors** consisting the **Strength - Mould** .

In Fig-3 The time-harmonic function $\bar{z}(t) \equiv \text{Wave}$, x is the amplitude, w the angular frequency, and Phase of $\bar{z}(t)$ to Be Determined by the number $n, n+1$ of Vertices .

The **complex - Vector** $\pm \bar{z}(t)$ uses the **Norm** $|\bar{z}| = 2 \cdot R$, and the number n , of the Space which Determines the Number of Knots and Sides of **Regular Polygons** as ,

1...The Space is consisted of 4 masses [99] which Produce the **Space - Wave** $= 2R \cdot e^{i \cdot n \omega t}$,

2...The Anti-Space is consisted of 5 masses Producing the **Anti-Space-Wave** $= -2R \cdot e^{i \cdot n \omega t}$

3...Sub-Space is consisted of 6 masses Producing the **Sub-Space-Wave** $= \pm 2R^{1/n} \cdot e^{i \cdot \theta/n}$

Remarks :

1.. The Real Shear -Stress that exists in the **Force-Carriers** Becomes from the Static Theory of Equivalent Tension (Moore's cycle) issues $(1) = \sigma = \sqrt{(\sigma_1 - \sigma_2)^2 + 4 \tau_{12}^2}$, and

$\sigma_{1,2} = (\sigma_1 + \sigma_2) / 2 \pm (\frac{1}{2}) \sqrt{(\sigma_1 - \sigma_2)^2 + 4 \tau_{yz}^2}$, instead of the classical case which is
 $\sigma = 2 \cdot \tau_{\max} = \sqrt{\sigma_1^2 + 4 \cdot \tau^2}$, . (2) . Because nature always follows the uniform tension so
it follows the (1) case. where exist the equation $\sigma = \sqrt{(\sigma_1 - \sigma_2)^2 + 4 \tau_{12}^2}$, and Stress $\sigma_{1,2}$
 $= (\sigma_1 + \sigma_2) / 2 \pm (\frac{1}{2}) \sqrt{(\sigma_1 - \sigma_2)^2 + 4 \tau_{yz}^2} = \frac{\sigma}{2} [\sqrt{5} + 1] = \sigma \cdot [\frac{\sqrt{5} + 1}{2}] \Rightarrow \text{Golden ratio for Stresses.}$

2.. The Bernoulli Astute about Stresses – Linear - Distribution is equally True for Bending too.
3.. In the case of concentrated Stresses (such as Closed or Open Conductors) , **The Doubling**

Of the main Stresses is as the Dublication of the Cube [126] and Trisection of angle [128] ,
where a **Right Angle Directional -Triangle** was of the relation $1 : 2 = 30^\circ / 60^\circ$, in **The case**
of the Isosceles Directional -Triangle $B_1 B_2$, is needed the relation $1 : 2 = 36^\circ / 72^\circ$,
and Observation is True , *And this Because of the Above Golden ratio Pattern .*

9d... The Complex - Force -Vectors from Golden-Ratio Stresses . [92 -114]

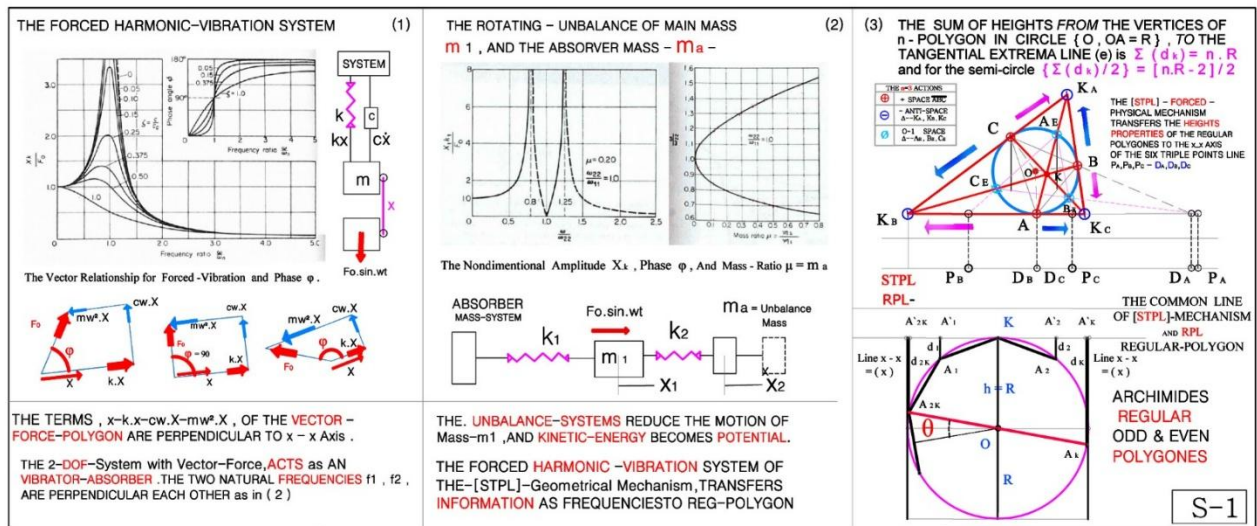


Figure – 15 - The Complex -Force -Vector is $F_0 \cdot [\cos(\omega t) + \sin(\omega t)] = F_0 \cdot e^{i \cdot [\omega t]}$.

In-1- The Terms $= (m \ddot{x}, c \dot{x}, k x)$, of Vector Force Polygon are Perpendicular to y-y axis.
In Case of Complex -Force -Vector the Polygon becomes an **Absorber** which
Reduces the motion from Kinetic Energy to **Potential Energy** (a Magnetic Field) .

In-2- The 2-DOF System with Vector Force such that , $\omega_{11}^2 = \frac{k_1}{m_1}$, $\omega_{22}^2 = \frac{k_2}{m_2}$, acts
as an **Vibrator Absorber** , and reduces the motion of the mass m_1 to zero .The two
Natural Frequencies , f_1 , f_2 , are Perpendicular $[\perp]$ **each other** as the Diagrams.
This happens for every action on the **8.[a , b , c , d] Physical - Mechanisms** .

In-3- The{ STPL } - Six Triple Points Line Mechanism is formed from **Three Tangents**
from A ,B ,C Spaces on 2R - Circle . Since **Pascal-Desargues** lines meet on Markos
STPL Mechanism lines ,**Then transfer the Properties & Information** (f_1 , f_2 , , f_n)
From The **Complex-Vector-Absorber** $= \overline{2R}$, $\rightarrow$ **To The Vector-Force-Polygons** $\leftarrow$

Remarks :

- 1.. **The n-Knots** Figures is any N-DOF System , which **Characteristic equation** results to an Algebraic equation of 3-n Degree , with main equation of motion $[\lambda.M + K] X = 0$
- 2.. **The n-Knots** Figures exist in Figures where the **Vector - Force - Polygons** have the same number **n** , as the in Figures . In (3) The Circularly- **Charged Breakages** are the three- **3** elements $[\Sigma = n \cdot h] \Rightarrow [s^2 = \oplus, 2s^2 = \emptyset, -s^2 = \ominus]$ which formulate the **3-Knots** Figures. This **{STPL}-Mechanism** of **3** elements ($n=3$) determinates the Mould which denotes the number **n** , of the Spaces for **Elementary-Particles Vector**.
- 3.. **The case of {TCIC}** $\equiv [\Sigma = n \cdot h] \Rightarrow 8 \times (1)$ which is the **Physical Mechanism** , gives the **n-masses** of Atom`s & Compounds Determining the **Atoms Complex-Vector-Response**.
- 4.. **The case of Dublication of Energy** as Total Energy $2.E = \text{Spin} = \check{S} = \vec{w} \cdot \vec{B} = \frac{\vec{c}}{r} \cdot \vec{B} = \vec{c} \cdot \left| \frac{\vec{B}}{r} \right|$
Results to $\vec{c} = \bar{c}_p = \left| \frac{2.r.\vec{E}}{B} \right|$ & The Centripetal velocity $\bar{c}_T \equiv \vec{R}\vec{O} \equiv |\vec{v}| = \frac{\sigma}{2} [1 + \sqrt{5}] \equiv \sigma.\Phi$,
 From equation $F = \sigma \cdot A = \left[\frac{2\pi r f}{\Phi} \right] \cdot A = w \cdot r \cdot \left[\frac{A}{\Phi} \right] = v \cdot \left[\frac{A}{\Phi} \right]$ is seen that any loaded **Area A** or **Conductor** $\vec{v}$, is related to the **Golden Ratio Pattern** Φ , either in **Plane or in Vector** .
 When Energy is in concentrated Stresses (such as Closed or Open Conductors) , then **The Doubling of the main Stresses** is as the Dublication of the Cube [126] , and The **Trisectioning of the main Stresses** is as the Trisection of angle [128] , where

THE NEEDED MOULDS

- a) is an **Isosceles Right Angle Directional -Triangle** which **Hypotenuse** lies on the Diameter of the circle with relation of Base-Angles $1 : 2 = 45^\circ / 90^\circ$, & it is the **Material-Mould for the Squaring The circle** & Energy-circle [124].
- b) is an **Isosceles Directional - Triangle** which **Base** is a **Chord** of the circle with relation of Base - Angles $2 : 1 = 72^\circ / 36^\circ$, and it is the , **Material-Mould for Constructing the Regular Polygons of the circle** . & The Polygons of the Energy-circle [130] .
- c) is an **Right Angle Directional - Triangle** which **Hypotenuse** lies on the Diameter of the circle with relation of Base - Angles $1 : 2 = 30^\circ / 60^\circ$, and it is the **Material-Mould for (1) The Dublication of the Cube** $\leftarrow$
(2) The Trisection of any Angle $\leftarrow$
 & **(1) The Dublication of the Energy - Cube** [126] .
(2) The Trisection of Energy - Octava . [128] .

The Golden ratio Φ on Segment AB , is at Point C Between A and B , while on the

Material Point $[\oplus \cup \ominus]$ is on Principal Stress σ as frequency $f_n \equiv \left[\frac{1+\sqrt{5}}{2} \right] \frac{\sigma}{2\pi r} \equiv \left[\frac{n\sigma}{8r^2} \right] \cdot \bar{B} \equiv \frac{E}{h}$

i.e. is the **Frequency f_n or Stress σ in Material - Point** and in cave 10^{-62} m & also as **The-Golden-ratio Pattern IN** & Under - **Planck`s-level** 10^{-35} m .

The Work $\equiv$ motion , in Material-Point is Produced from the frequencies f_N , &

$$f_N [S \equiv f_{1=n}, f_2, f_3, f_R = w^2] = n \frac{(1+\sqrt{5})\sigma}{4\pi r} = \frac{n\sigma.\bar{B}}{8r^2} \leftarrow \text{The meter of Stationary motion .}$$

E..THE GALOIS – CAUSS CONSTRUCTIBILITY OF THE REGULAR–POLYGONS

Gaus Galois Constructibility of Regular Polygons



★ AI Overview

A regular n -sided polygon can be constructed with a compass and straightedge if and only if n is the product of a power of 2 and any number of distinct Fermat primes ($2^{2^k} + 1$). Carl Friedrich Gauss proved the sufficiency in 1796, and Pierre-Laurent Wantzel proved necessity using modern algebraic field theory. [Wikipedia +2](#)



The Gauss-Wantzel Theorem

- **Constructibility condition:** $n = 2^k p_1 p_2 \cdots p_m$.
- **Fermat primes (p_i):** Prime numbers of the form $p = 2^{2^k} + 1$.
- **Known Fermat primes:** 3, 5, 17, 257, and 65537.
- **Examples of constructible polygons:** Triangle ($n = 3$), pentagon ($n = 5$), heptadecagon ($n = 17$), and 15-gon (3×5).
- **Examples of non-constructible polygons:** Heptagon ($n = 7$), nonagon ($n = 9$), and undodecagon ($n = 11$). [Wikipedia +4](#)

Connection to Field Extensions and Galois Theory

- **Cyclotomic fields:** Constructing a regular n -gon is equivalent to building a primitive n -th root of unity ζ_n in the complex plane.
- **Degree of extension:** The extension field $\mathbb{Q}(\zeta_n)$ over $\mathbb{Q}$ has a degree given by Euler's totient function $\varphi(n)$.
- **Power of 2 requirement:** Ruler-and-compass operations restrict field extension degrees to powers of 2. Thus, $[\mathbb{Q}(\zeta_n) : \mathbb{Q}] = \varphi(n) = 2^k$.
- **Galois group symmetry:** The Galois group $\text{Gal}(\mathbb{Q}(\zeta_n)/\mathbb{Q})$ is abelian and isomorphic to $(\mathbb{Z}/n\mathbb{Z})^\times$, breaking down cleanly via nested square roots only when $\varphi(n)$ is a power of 2. [YouTube · Richard E Bo... +1](#)

The Judgment regarding The Correctness of The Constructibility , is left Exclusively to The Dialectic Logic of Nature And to No other Logic or Reason
→ Markos Georgallides 26 / 09 / 2026 ←

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Article [124] is Dedicated to the **National Technical University of Athens** , NTUA , from where I Graduated and Acquired the Path Knowledge of the Truths of Nature..

Το Άρθρο [124] είναι Αφιερωμένο στο Εθνικό Μετσόβιο Πολυτεχνείο (ΕΜΠ) , από όπου Αποφοίτησα και Απέκτησα την Ατραπό της Γνώσης των Αληθειών της Φύσης.

Markos Georgallides comes from Cyprus and currently he resides in the City Larnaca , Cyprus after being expelled from his Home Town **Famagusta - Varosha** , by The Barbaric Turks in August 1974 . He works as a consultant Civil and Architect Engineer having his own Business . He is also The author of numerous scholarly Articles focusing on **Euclidean** and **Material Geometry** , and mathematical To Physics and Mechanics related Subjects . He obtained his degree from the Athens , National Technical , Polytechnic University [NATUA] Athens- Greece , and subsequently studied in Germany , Math Theory of Photoelasticity .