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ANALYSIS OF OPTION PRICING MECHANISMS IN FINANCIAL MARKETS USING THE BLACK-SCHOLES MODEL AND SNEDECOR'S F-DISTRIBUTION

By Author(s):

Azor, Promise Andaowei*
Department of Mathematics & Statistics, Federal University, Otuoke, Nigeria,

Abstract

The success of any investment largely depends on the valuation of options, which plays a pivotal role in shaping the financial strategies of investors. The Black-Scholes (B-S) equation remains a foundational mathematical tool for estimating stock option prices. This study examines the Black-Scholes model for European call options alongside Snedecor's F-distribution to evaluate option pricing on the share prices of Fidelity and Access Banks. Closed-form solutions for call option prices were obtained for two distinct expiration dates. The variation in call option prices between these dates provides valuable insights into the market's expectations regarding future movements of the underlying securities.

Additionally, hypothesis testing was conducted and accepted for both banks, revealing statistically significant differences in the variances of call option prices across expiration dates. The analysis yielded variances of 0.8954 for Fidelity Bank and 0.9746 for Access Bank, indicating that higher variance implies greater potential for price fluctuation over time. Hence, Fidelity Bank, with the lower variance, offers better precision for informed investment decisions.

Furthermore, a theoretical proposition was formulated and validated to analyze prospective price changes and support strategic decision-making. The study also considered the means and standard deviations of projected future prices, offering practical implications for understanding investment returns within capital markets.

Keywords:

Black-Scholes, Call Option, Financial Market, European Call Option, Put Option. Stock Market

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* Corresponding Email: azor.promise@gmail.com

1. INTRODUCTION

Black-Scholes is a pricing model used to determine the fair price or theoretical value for a Call or a Put Option based on six variables such as volatility, type of option, underlying stock price, time, strike price, and risk-free rate. The quantum of speculation is more in case of stock market derivatives, and hence proper pricing of options eliminates the opportunity for any arbitrage. There are two important models for option pricing – Binomial Model and Black-Scholes Model. The model is used to determine the price of a European call option, which simply means that the option can only be exercised on the expiration date. The Black-Scholes model is used to calculate the theoretical price of European put and call options, where an option is "a contract for the right to buy and sell shares at a later date or within a certain period at a particular price" (Cambridge Online Dictionary 2017a). Therefore, they assumed some features of the financial market, including: European-style options: The model supposes European-style options. Those can only be exercised on the expiration date. With American-style options it is possible to exercise the option at any time during the life of the option.

However, the use of Snedecor's F-distribution or simply F-distribution is a continuous probability distribution used in statistiiics. It is used test the significance of differences between variances of two populations. The distributions named after George W. Snedecor, an American statistician who played a key role in its developments, [1]. So, in the case of this project is used to determine the variances of two Call option prices at different expiration dates which will give option trader insight into the volatility of the underlying security.

Nevertheless, analytical approach of solving Black-Scholes equation for option pricing is not a difficult one. It becomes multifarious when some levels of probability distributions like Senedecor's F- distribution are considered to assess Call option prices. This difficulty depends when the analytical approach is being sought; which is the novelty of this project. This paper extends the work of [2] on their analysis of valuing share price of Fidelity and Access banks. [3] was of the opinion that the cost of the option lies on the underlying asset, which is usually a stock, commodity, currency or an index. The holder has the right but cannot be compelled to buy, for call option where European put option involves the ability to sell an asset for a certain charge at a prescribed date in the future. Options are known as "*in- the money*", "*at- the money*", or "*out- of the money*". If S is a stock price and K is the strike price, a call option is in- the money as soon as $S > K$, at- the money when $S = K$, and out -of –the- money when $S < K$. A put option is in the money as soon as $S < K$, at- the -money once $S = K$, and out- of- the money once $S > K$. Obviously, an option is exercised only when it is in- the money. In the nonappearance of transactions costs, an in-the money option is always exercised on the expiration date if it has not been exercised earlier as seen by [4]. For instance,[5] analyzed B-S formula for the valuation of European options; Hermite polynomials were applied. They concluded that BS formula can easily be achieved devoid of the use of partial differential equation. In another study of BS, [6] considered the B-S terminal value problem and observed that their proposed method is better, simple than the previous methods. In the work of [7], time varying factor were incorporated in the explicit formula for different aspect of options with the aim of providing exact solution for dividend paying equity of option. In considering the stability of stock market price of stochastic model, [8] applied Crank-Nicolson numerical scheme to B-S model. The results showed stock prices being stable and its increasing rate of stock shares was obtained. Not quite long, [9]

investigated the variation of stock market price using B-S PDE. The convergence to equilibrium of growth rate and sufficient conditions for stability was achieved. However, [10] studied Black-Scholes model because of its biasness in mispricing options. They established a new technique of assessing pricing effects on the premise to reduce pricing bias. In recent years the fields of Quantum Economics and Quantum In his work, [11], he determined the complete symmetry analysis of the one-dimensional Black-Scholes equation and constructed invariant solutions for some examples. Financial derivatives refer to contracts whose values depend on the value of the underlying asset, and are an important part of financial innovation tools [12]. On the other hand, the suitable option pricing model can perform risk assessment on secured loans and pension insurance [12]. However, it is practically impossible to execute the strict assumption of B-S model in real markets which are usually not perfectly liquid
[15] and does not conform to the actual situation of option transactions. Many scholars have found that there are discrepancies or errors between the Option prices obtained from the B-S model and the market prices. The modification and expansion of the Black-Scholes model have important practical significance for the construction of a practical and scientific sound option pricing method, and at the same time can provide an important theoretical basis for the management, avoidance and control of financial risks [12]. The current forms of modifications are mainly based on revise and expand the B-S model from the basic model and price parameters. From the perspective of the basic model, [11] respectively proposed the jump diffusion model and the GARCH model.

Furthermore, [13] concluded that since pricing models generally require the assumption that stock prices are described by continuous-time stochastic processes, the time-continuous trading is easy to conceive theoretically, and it is practically impossible to execute in real markets. One reason is because real markets are not perfectly liquid and purchase or sell any amount of an asset would change the asset price drastically. A realistic hedging strategy needs to consider trading that happens at discrete instants of time. His paper focused on the impact and effect due to temporal discretization on the pricing partial differential equation (PDE) for European options. Two different aspects of temporal discretization were considered and used to derive the modification or correction source terms to the continuous pricing PDE. First the finite difference discretization of the standard Black–Scholes PDE and its modification due to discrete trading. Second the discrete trading led to a discrete time re-balancing strategy that only cancels risks on average by using a discrete analogy of the stochastic process of the underlying asset. In both cases high order terms in the Taylor series expansion were used and the respective correction source terms were derived. The B-S model is a commonly used Option pricing model, which assumes that the volatility and the interest rate are constants. However, the empirical analysis shows that the Option prices obtained from the B-S model are quite different from the market prices. The traditional improvement to the B-S model is to replace the volatility constants or interest rate constants with variables.

The minimal price of an Option under transaction costs is obtained [14] considered analytic formula of Black-Scholes model for share prices of Fidelity and Access banks which gave closed form prices of Call Options. The simulation results show: the higher the share price determines the value of call option prices, an increase on the maturity days dominantly increases the value of Call Option for both Fidelity, Access and their future merging, merging of the two banks improved on the value of call option prices, Fidelity bank has a good maximum value of call option prices during the period of investments. However, reviews show that Black-Scholes have copious applications as seen in the literature. This present project is aimed at pricing European options with Snedecor's F-distribution on the analysis of Share prices for capital market investments as this will add values in this dynamic area of statistical finance. Nevertheless, the knowledge gap in literature reviewed shows that none of scholars considered Call and Snedecor's F-distribution in analyzing share prices of Fidelity and Access banks, and besides no stating and proving of proposition were considered.

2. Purpose of this Paper

The purpose of this paper is to determine the effects of the Black- Scholes model and Snedecor's F-distribution in Pricing Option.

3. Mathematical Preliminaries

Here we present some definitions as foundations of this mathematical finance models.

Definition 3.1: Probability space: This is a triple $(\Omega, F, \tilde{A})$ where Ω represents a set of sample space, F represents a collection of subsets of Ω, while $\wp$ is the probability measure defined on each event $A \in F$. The collection F is a σ-algebra or σ-field such as $\Omega \in F$ and F is closed under the arbitrary unions and finite intersections. Hence it is called probability measure when the following condition holds.

(i) $P(A) \geq 0 \ \ for \ all \ A \subset \Omega$ (1)

(ii) $P(\Omega) = 1$ (2)

(iii) $A, B \subset \Omega, \ A \cap B = \phi \ then \ P(A \cup B) = P(A) + P(B)$ (3)

Definition 3.2. Normal Distribution: A normal distribution function is a peculiar distribution in probability theory and is usually used for modeling asset returns. A normal distribution is used in the Black-Scholes Partial differential equation to value European options. A normal distribution depends on two parameters.

- Mean, $\mu \in \mathbb{R}$, is the expectation of a random variable normal distribution.
- Variance, $\sigma^2 > 0$, deals with the magnitude of the spread from the mean.

In Black-Scholes formula, normal distributions are used. The cumulative distribution, usually denoted as $\phi(X)$, is the probability that X will be equal to or less than x ,expressed as $F_x(x) = P(X \leq x)$. A standard normal cumulative distribution function is defined as.

$$\phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{t^2}{2}} dt \qquad (4)$$

A normal distribution is a symmetric distribution, which means that it touches around a vertical axis of symmetry. Obviously, there is a connection between any given points with same distance to the vertical axis. This relationship is defined in equation (5) .

$$\phi(x) = 1 - \phi(-x) \qquad (5)$$

Definition 3.3. A σ-algebra is a set F of subsets of Ω with the following axioms:

(i) $\phi, \Omega \in F$ (6)

(ii) If $A \in F, then\ A^c \in F$ (7)

(iii) If $A_1, A_2, \ldots, \in F, then \bigcup_{k=1}^{\infty} A_k, \bigcap_{k=1}^{\infty} A_k \in F$ (8)

Clearly $A^c := \Omega - A$ is the complement of A.

Definition 3.4. If F is a σ-algebra in Ω, then Ω is called a measurable space and the members of F are called the measurable sets in Ω.

Definition 3.5. Let (Ω, M) be a measurable space A map $\mu : M \to \square = [0, \infty) \cup \{\infty\}$ is called a measure provided that

(i) $\mu(\phi) = 0$ (9)

(ii) $\mu\left(\bigcup_{n=1}^{\infty} A_n\right) = \sum_{n=1}^{\infty} \mu(A_n)$ (10)

Definition 3.6. Stochastic process: A stochastic process $X(t)$ is a relation of random variables $\{X_t(\gamma), t \in T, \gamma \in \Omega\}$, i.e., for each t in the index set T, $X(t)$ is a random variable. Now we understand t as time and call $X(t)$ the state of the procedure at time t. In view of the fact that a stochastic process is a relation of random variables, its requirement is similar to that for random vectors.

It can also be seen as a statistical event that evolves time in accordance to probabilistic laws. Mathematically, a stochastic process may be defined as a collection of random variables which are ordered in time and defines at a set of time points which may be continuous or discrete.

Definition 3.7. A stochastic process whose finite dimensional probability distributions are all Gaussian. (Normal distribution).

Definition 3.8. Random Walk: There are different methods to which we can state a stochastic process. Then relating the process in terms of movement of a particle which moves in discrete steps with probabilities from a point $x = a$ to a point $x = b$. A random walk is a stochastic sequence $\{S_n\}$ with $S_0 = 0$, defined by

$$S_n = \sum_{k-1}^{n} X_k \quad (11)$$

where X_k are independent and identically distributed random variables

Definition 3.9: Stochastic Differential Equation (SDE)

Let $S(t)$ be the price of some risky asset at time t ,and μ ,an expected rate of returns on the stock and dt as a relative change during the trading days such that the stock follows a random walk which is govern by a stochastic differential equation.

$$dS(t) = \alpha S(t)dt + \sigma S(t)dW_t \quad (12)$$

Where, α is drift and σ the volatility of the stock, W_t is a Brownian motion or Wiener's process on a probability space $(\Omega, \xi, \wp), \xi$ is a σ − algebra generated by $W_t, t \geq 0$.

Theorem 1.1: **(Ito's formula)** Let $(\Omega, \beta, \alpha, F(\beta))$ be a filtered probability space $X = \{X, t \geq 0\}$ be an adaptive stochastic process on $(\Omega, \beta, \alpha, F(\beta))$ possessing a quadratic variation (X) with SDE defined as:

$$dX(t) = g(t, X(t))dt + f(t, X(t))dW(t)$$

$t \in \Re$ and for $u = u(t, X(t) \in C^{1\times 2}(\Pi \times \Pi))$

$$du(t, X(t)) = \left\{\frac{\partial u}{\partial t} + g\frac{\partial u}{\partial x} + \frac{1}{2}f^2\frac{\partial^2 u}{\partial x^2}\right\}d\tau + f\frac{\partial u}{\partial x}dW(t)$$

Using theorem 3.1 and equation (3.2.12) comfortably solves the SDE with a solution given below:

$$S(t) = S_0 \exp\left\{\sigma dW(t) + \left(\alpha - \frac{1}{2}\sigma^2\right)t\right\}, \forall t \in [0,1] \qquad (13)$$

3.2.2 Derivation of Black-Scholes (B-S) Equation

The B-S Equation used no-arbitrage argument to explain a partial differential equation which governs the growth of the option price with esteem to the expiration and cost of the fundamental Asset.

Suppose we have an option whose worth V(S,t) depends only on S and t. Assume also that the asset price is varied by a small amount of change dS, then the function V will also be affected. By means of Itoˆ lemma

$$dV = \sigma S\frac{\partial V}{\partial S}dx + \left(\mu S\frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2 S^2\frac{\partial^2 V}{\partial S^2} + \frac{\partial V}{\partial t}\right)dt \qquad (14)$$

According to [15], the value of one portfolio having one stock can be expressed with the function $V(S,t)$.

$$\pi = V - \Delta S \qquad (15)$$

The change in the portfolio at time dt in (3) is given by

$$d\pi = dV - \Delta S \qquad (16)$$

Putting (1), (2) into (4), we find that π follows random walk given by.

$$d\pi = \sigma S\left(\frac{\partial V}{\partial S} - \Delta\right)dx + \left(\mu S\frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2 S^2\frac{\partial^2 V}{\partial S^2} + \frac{\partial V}{\partial t} - \mu\Delta S\right)dt$$

To eliminate the random component in this random walk, let

$$\Delta = \frac{\partial V}{\partial S} \qquad (17)$$

Note that Δ is the value of $\frac{\partial V}{\partial S}$ at the start of the time step dt. This results in a portfolio whose increment is wholly deterministic so that

$$d\pi = \left(\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} \right) dt \qquad (18)$$

Now that the portfolio is riskless it should earn riskless return. The change in the portfolio at time dt becomes (after substituting (1) and (2) into (4) and dividing through by dt)

$$d\pi = r\pi\, dt = \left(\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} \right) dt$$

and

$$r\left(V - \Delta S\right) dt = \left(\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} \right) dt$$

which implies that

$$rV - rS\frac{\partial V}{\partial S} = \frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2}$$

This gives the solution

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0 \qquad (19)$$

This is the Black-Scholes partial differential equation as established.

Here, we transform Black-Scholes PDE into heat equation, apply the solution through integration, and convert back to the original parameters. To get rid of the S and S^2 terms in the Black-Scholes PDE. The transformations are as follows:

$$\left.\begin{aligned} x &= \ln\frac{S}{K}, \text{ so that } S = Ke^x \\ \tau &= \frac{\sigma^2 K}{2}\left(T - t\right) \text{ so that } t = T - \frac{2r}{\sigma^2} \\ U\left(x,\tau\right) &= \frac{1}{K}V\left(S,t\right) = \frac{1}{K}V\left(\mathrm{K}e^x, \mathrm{T} - 2\mathrm{r}/\sigma^2\right) \end{aligned}\right\} \qquad (20)$$

Applying chain rule to the partial derivatives in the Black-Scholes PDE; (19) yields

$$\left.\begin{aligned}
\frac{\partial V}{\partial t} &= K\frac{\partial U}{\partial \tau}\frac{\partial \tau}{\partial t} = \frac{-K\sigma^2}{2}\frac{\partial U}{\partial \tau},\\
\frac{\partial V}{\partial S} &= K\frac{\partial U}{\partial x}\frac{\partial x}{\partial S} = \frac{K}{S}\frac{\partial U}{\partial x} = e^{-x}\frac{\partial U}{\partial x},\\
\frac{\partial^2 V}{\partial S^2} &= -\frac{K}{S^2}\frac{\partial U}{\partial x} + \frac{K}{S}\frac{\partial}{\partial S}\left(\frac{\partial U}{\partial x}\right)\\
&= -\frac{K}{S^2}\frac{\partial U}{\partial x} + \frac{K}{S}\frac{\partial}{\partial x}\left(\frac{\partial U}{\partial x}\right)\frac{\partial x}{\partial S}\\
&= -\frac{K}{S^2}\frac{\partial U}{\partial x} + \frac{K}{S^2}\frac{\partial^2 U}{\partial x^2}\\
&= \frac{e^{-2x}}{K}\left(\frac{\partial^2 U}{\partial x^2} - \frac{\partial U}{\partial x}\right)
\end{aligned}\right\} \tag{21}$$

Putting (21) in the Black-Scholes PDE in (19) to obtain

$$\frac{-K\sigma^2}{2}\frac{\partial U}{\partial \tau} + rKe^{x}e^{-x}\frac{\partial U}{\partial x} + \frac{1}{2}\sigma^2K^2e^{2x}\frac{e^{-2x}}{K}\left(\frac{\partial^2 U}{\partial x^2} - \frac{\partial U}{\partial x}\right) - rU = 0$$

By simplifying, gives;

$$-\frac{\partial U}{\partial \tau} + (k-1)\frac{\partial U}{\partial x} + \frac{\partial^2 U}{\partial x^2} - kU = 0 \tag{22}$$

Where $k = \frac{2r}{\sigma^2}$. The coefficients of this PDE does not involve x or τ.The boundary condition for V *is* $V(S_{T,}\ T) = (S_T - K)^+$ .From (1.1), when $t = T$ *and* $S_t = S_T$ *we have that* $x = \ln\frac{S_T}{K}$ which we write as $x_{T,}$ and that $\tau = 0$. Hence the boundary condition for U is :

$$U_0(x_T) = U(x_T, 0) = \frac{1}{K}V(S_T - K)^+ = \frac{1}{K}\left(Ke^{x_T} - K\right)^+ = \left(e^{x_T} - 1\right)^+$$

We make additional transformation

$$W(x,\tau) = e^{\alpha x + \beta^{2\tau}}U(x,\tau) \tag{23}$$

where $\alpha = \frac{1}{2}(k-1)$ *and* $\beta = \frac{1}{2}(k+1)$. This will convert (22) into the heat equation. The partial derivatives of U in terms of W are as follows:

$$\left.\begin{aligned}\frac{\partial U}{\partial \tau} &= e^{-\alpha x-\beta^2\tau}\left(\frac{\partial W}{\partial \tau} - W(x,\tau)\beta^2\right)\\ \frac{\partial U}{\partial x} &= e^{-\alpha x-\beta^2\tau}\left(\frac{\partial W}{\partial x} - \alpha W(x,\tau)\right)\\ \frac{\partial^2 U}{\partial x^2} &= e^{-\alpha x-\beta^2\tau}\left(\alpha^2 W(x,\tau) - 2\alpha\frac{\partial W}{\partial x} + \frac{\partial^2 W}{\partial x^2}\right)\end{aligned}\right\} \quad (24)$$

Substitute these derivatives (24) to obtain;

$$\beta^2 W(x,\tau) - \frac{\partial W}{\partial \tau} + (k-1)\left[-\alpha W(x,\tau) + \frac{\partial W}{\partial x}\right] + \alpha W(x,\tau) - 2\alpha\frac{\partial W}{\partial x} + \frac{\partial^2 W}{\partial x^2} - kW(x,\tau) = 0$$

Which simplifies to heat equation

$$\frac{\partial W}{\partial \tau} = \frac{\partial W^2}{\partial x^2} \quad (25)$$

From (4) the boundary condition for $W(x,\tau)$ is;

$$W_0(x_T) = W(x_T,0) = e^{\alpha x_T}U(x_T,0) \quad (26)$$

$$\left(e^{(\alpha+1)x_T} - e^{\alpha x_T}\right)^+ = \left(e^{\beta x_T} - e^{\alpha x_T}\right)^+$$

Since $\beta = \alpha + 1$. The transformation from V *to* W becomes

$$V(S,t) = \frac{1}{K}e^{-\alpha x-\beta^2 T}W(x,\tau) \quad (27)$$

To obtain Black-Scholes Call price; since $W(x,\tau)$ follows heat equation, it has a solution given by:

$$\left.\begin{aligned}&\int_{-\infty}^{\infty} u(x-\xi)\delta(\xi)\,d\xi\\ u(x,\tau) &= \int_{-\infty}^{\infty} u(x-\xi)\,\mathrm{u}_0(\xi)\,\mathrm{d}\xi\\ &= \int_{-\infty}^{\infty} \frac{1}{\sqrt{4\pi\tau}} e^{-(x-\xi)^2\,\mathrm{u}_0(\xi)\,\mathrm{d}\xi}\end{aligned}\right\}$$

with initial value

$$\mathrm{u}(x,0) = \int_{-\infty}^{\infty} \delta(x-\xi)\,\mathrm{u}_0(\xi)\,\mathrm{d}\xi = u_0(x)$$

With boundary condition given by (27). Hence the solution is

$$W(x,\tau) = \frac{1}{\sqrt{4\pi\tau}} \int_{-\infty}^{\infty} e^{-(x-\xi)^2/4\tau} W_0(\xi)\, d\xi$$

$$= \frac{1}{\sqrt{4\pi\tau}} \int_{-\infty}^{\infty} e^{-(\xi-x)^2/4\tau} \left(e^{\beta\xi} - e^{\alpha\xi}\right)^+ d\xi$$

Making change of variable $z = \frac{\xi - x}{\sqrt{2\tau}}$ so that $\xi = \sqrt{2\tau}\, z + x$ and $d\xi = \sqrt{2\tau}\, dz.$

$$W(x,\tau) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left(-\frac{1}{2} z^2\right) \times \exp\left(\beta\left[\sqrt{2\tau}\, z + x\right] - \alpha\left[\sqrt{2\tau}\, z + x\right]\right)^+ dz \qquad (28)$$

Note that the integral is non-zero only when the second exponent is greater than zero, that is, when $\beta\left[\sqrt{2\tau}\, z + x\right] > \alpha\left[\sqrt{2\tau}\, z + x\right]$ which is identical to $z > \frac{-x}{\sqrt{2\pi}}$. We now break up the integral into two pieces

$$W(x,\tau) = \frac{1}{\sqrt{2\pi}} \int_{-x/\sqrt{2\tau}}^{\infty} \exp\left(-\frac{1}{2} z^2\right) \exp\left(\beta\left[\sqrt{2\tau}\, z + x\right]\right) dz - \frac{1}{\sqrt{2\pi}} \int_{-x/\sqrt{2\tau}}^{\infty} \exp\left(-\frac{1}{2} z^2\right) \exp\left(\alpha\left[\sqrt{2\tau}\, z + x\right]\right) dz$$

$$= I_1 - I_2.$$

Completing the square in the first integral I_1 .The exponent in the integrand is

$$-\frac{1}{2} z^2 + \beta\sqrt{2\tau}\, z + \beta x = -\frac{1}{2}\left(z - \beta\sqrt{2\tau}\right)^2 + \beta x + \beta^2\tau.$$

The first integral yields

$$\mathrm{I}_1 = e^{\beta x + \beta^2\tau} \frac{1}{\sqrt{2\pi}} \int_{-x/\sqrt{2\tau}}^{\infty} e^{-\frac{1}{2}\left(z-\beta\sqrt{2\tau}\right)^2} dz$$

Making transformation $y = z = \beta\sqrt{2\tau}$ such that the integral gives

$$\mathrm{I}_1 = e^{\beta x + \beta^2\tau} \frac{1}{\sqrt{2\pi}} \int_{-x/\sqrt{2\tau} - \beta\sqrt{2\tau}}^{\infty} e^{-\frac{1}{2} y^2} dy$$

$$= e^{\beta x + \beta^2\tau} \left(1 - \Phi\left(-\frac{x}{\sqrt{2\tau}} - \beta\sqrt{2\tau}\right)\right)$$

$$= e^{\beta x + \beta^2\tau} \Phi\left(\frac{x}{\sqrt{2\tau}} + \beta\sqrt{2\tau}\right).$$

The second integral is identical, except that β is replaced with α. Hence;

$$I_2 = e^{\alpha x + \alpha^2\tau} \Phi\left(\frac{x}{\sqrt{2\tau}} + \alpha\sqrt{2\tau}\right)$$

Re*call that* $x = \ln\frac{S}{K}$, $k = \frac{2\tau}{\sigma^2}$, $\alpha = \frac{1}{2}(k-1) = \frac{r - \sigma^2/2}{\sigma^2}$, $\beta = \frac{1}{2}(k+1) = \frac{r + \sigma^2/2}{\sigma^2}$, and $\tau = \frac{1}{2}\sigma^2(T-t)$

Consequently, we have that

$$\frac{x}{\sqrt{2\tau}} + \beta\sqrt{2\tau} = \frac{\ln\frac{S}{K} + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} = d_1$$

And that

$$\frac{x}{\sqrt{2\tau}} + \alpha\sqrt{2\tau} = d_1 - \sigma\sqrt{T-t} = d_2$$

Hence the first integral becomes

$$I_1 = \exp\left(\beta x + \beta^2\tau\right)\Phi\left(d_1\right)$$

The second integral is identical except that β is replaced by α and involves d_2 instead of d_1

$$I_2 = \exp\left(\alpha x + \alpha^2\tau\right)\Phi\left(d_2\right)$$

The solution is therefore

$$\begin{aligned} W(x,\tau) &= I_1 - I_2 \\ &= e^{\beta x + \beta^2\tau}\Phi(d_1) - e^{\alpha x + \alpha^2\tau}\Phi(d_2) \end{aligned} \quad (29)$$

The solution in (29), expressed in terms of I_1 and I_2 ,is the solution for $W(x,\tau)$.To obtain the solution for thecall price $V(S_t,t)$

We must use (27) and transform the solution in (9) back to V. From (27) and (29)

$$\begin{aligned} V(S,t) &= Ke^{-\alpha x - \beta^2\tau}W(x,\tau) \\ &= Ke^{-\alpha x - \beta^2\tau}\left[I_1 - I_2\right] \end{aligned} \quad (30)$$

The first integral in (30) is

$$\begin{aligned} Ke^{-\alpha x - \beta^2\tau}e^{\beta x + \beta^2\tau}\Phi(d_1) &= Ke^{(\beta-\alpha)x}\Phi(d_1) \\ &= S\Phi(d_1) \end{aligned} \quad (31)$$

Since $\beta - \alpha = 1$. The second integral in equation (1.11) becomes

$$\begin{aligned} Ke^{-\alpha x - \beta^2\tau}e^{\alpha x + \alpha^2\tau}e^{\alpha x + \alpha^2}\Phi(d_2) &= Ke^{\left(\alpha^2 - \beta^2\right)\tau}\Phi(d_2) \\ &= Ke^{-r(T-t)\tau}\Phi(d_2) \end{aligned} \quad (32)$$

Since $\alpha^2 - \beta^2 = -\frac{2\tau}{\sigma^2}$

, Combining the terms in (31) and (32) produces the Black-Scholes Call price in equation as follows for both options:

$$\left.\begin{aligned} C &= SN(d_1) - Ke^{-rt}N(d_2) \\ d_1 &= \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \\ d_2 &= d_1 - \sigma\sqrt{T} \end{aligned}\right\} \qquad (33)$$

where C is Price of a call option, S is price of underlying asset, K is the strike price, r is the riskless rate, T is time to maturity, σ^2 is variance of underlying asset, σ is standard deviation of the (generally referred to as volatility) underlying asset, and N is the cumulative normal distribution. Similarly, the formula for prices of European put option is given as

$$P = SN(d_1) - Ke^{-rt}N(d_2) \qquad (34)$$

where P is the price of a put option and the meaning of other parameters remain the same as in (1) [4].

The Black-Scholes model is made on seven assumptions:

i. The asset price has characteristics of a Brownian motion with μ and σ as constants.
ii. The transaction costs or taxes are not allowed.
iii. The entire securities are absolutely divisible.
iv. Dividend is not permitted during the period of the derivatives.
v. Unacceptable of riskless arbitrage opportunities.
vi. The security trading is constant.
vii. The option is exercised at the time of expiry for both call and put options.

3.3.3 Problem Formulation of Snedecor's F-Distribution

However, assuming there are N stocks in the capital market. Let S_i, $i = 1, 2, \ldots, N$ be the initial stock prices for T trading days. Considering the two Call option prices of sizes n_1 and n_2 via Fidelity and Access bank share prices whose variances are σ_1^2 and σ_2^2 respectively.

Now, let $S_1^2 = \frac{1}{n}\sum_{i=1}^{n}(X_{1i} - \bar{X})^2$ and $S_2^2 = \frac{1}{n}\sum_{i=1}^{n}(X_{2i} - X)^2$ represent the variances of the first Call and second Call options. For simplicity, the F-distribution is defined as the ratio of)(^{2} variates which gives:

$$F = \frac{x^2_{n1}}{n_1} \Big/ \frac{x^2_{n2}}{n_2} = \frac{(n_1 - 1)S_1^2}{\sigma_1^2} / n_1 \Big/ \frac{(n_2 - 1)S_2^2}{\sigma_2^2} / n_2 = \frac{S_1^2}{\sigma_1^2} \Big/ \frac{S_2^2}{\sigma_2^2} \qquad (3.2.35)$$

Since Black-Scholes equation is based on seven assumptions,

3.3.4 Properties of F-Distribution

i. F-distribution has two parameters, i.e., $V_1 = (n_1 - 1)$ *and* $V_2 = (n_2 - 1)$

ii. the mean of F-variate with V_1 *and* V_2 degree of freedom is $\frac{V_2}{V_2 - 2}$

-the standard error is given as:
and

$$S.E = \left(\frac{V_2}{V_2 - 2}\right)\sqrt{\frac{2(V_1 + V_2 - 2)}{V_1(V_2 - 4)}}$$

if $V_2 > 2$, the mean will exist

if $V_2 > 4$, the standared error will exist and if the mean is greater than 1.i.e the mean>1

i. the random variate F can take only positive values from $0 \text{ to } \infty$.

ii. the F-distribution approaches normal distribution for larger values of V_1 and V_2.

The above concepts are seen in the work of [16].

Also, from where the share price data of Fidelity and Access bank, PLC is derived. Following the method of Amadi et al. (2022). The best two Call prices are chosen in terms of minimum value criterion for Fidelity and Access banks.

$$\left.\begin{aligned} \beta_1 &= \min(\varphi_1, \varphi_2, \ldots, \varphi_n) \\ \beta_2 &= \min(\alpha_1, \alpha_2, \ldots, \alpha_n) \end{aligned}\right\} \qquad (36)$$

$$\left.\begin{aligned} \beta_3 &= \min(\varphi_1, \varphi_2, \ldots, \varphi_n) \\ \beta_4 &= \min(\alpha_1, \alpha_2, \ldots, \alpha_n) \end{aligned}\right\} \qquad (37)$$

4. Results

This Section presents the simulated results for the problem stated in Subsection 3 .3.3. The graphical results are implemented using Matlab programming software.

Table 1: The value of Share price of Fidelity Bank, PLC and and their respective differences with the following parameter values: $r = 0.03, \sigma = 0.25, k = 450$

Initial share price (S_0)	Call Option prices at time $t = 6$	Call Option prices at time $t = 12$	Differences
415	100.34	140.20	39.86
62	0.016	0.45	0.434
138	1.65	8.0033	6.3533
61	0.014	0.42	0.406
121	0.85	5.23	4.38
81	0.088	1.27	1.182
139	1.71	8.19	6.48
80	0.08	1.21	1.13
384	81.095	118.95	37.855

Table 2: The value of Share price of Access Bank, PLC and their respective differences with the following parameter values: $r = 0.03,\ \sigma = 0.25,\ k = 450$

Initial share price (S_0)	Call Option prices at time $t = 6$	Call Option prices at time $t = 12$	Differences In Call Option prices
410	97.12	137	39.88
80	0.081	1.21	120.919
126	1.049	5.98	4.931
79	0.075	1.15	1.075
98	0.27	2.54	2.27
92	0.19	2.028	1.838
127	1.093	6.13	5.037
91	0.18	1.95	1.77
378	77.57	114.1	36.53

4.2 Snedecor's F-Distribution for Hypothesis Testing

Applying Section 3.3.3 for Fidelity Call Option prices via Snedecor's F-distrbution

H_0: There is no significance difference between the variances of Call option prices at different expiration dates.

H_1: There is significance differences between the variances of Call option prices at different expiration dates

We shall reject H_0 Since the calculated: 0.8954 less than tabulated:15.5073, we accept H1 and conclude that there is significance difference between the variances of Call option prices at different expiration dates.

Similarly for Access bank:

H_0: There is no significance difference between the variances of Call option prices at different expiration dates.

H_1: There is significance difference between the variances of Call option prices at different expiration dates

We shall reject H_0 Since calculated: 0.9746 less than tabulated: 15.5073, we accept H1 and conclude that there is significance differences between the variances of Call option prices at different expiration dates.

Proposition 1: Suppose the trading f shares r adjustment f portfolios s allowed to take place; adjusting the Call Option prices Fidelity and Access bank based on (36) and (37) respectively. In order to recover the past trading of shares using the following share price dynamics as:

$$F_{BANK} = 0.42e^{0.08t}dt \qquad (38)$$

$$A_{ACCESS} = 1.15e^{0.18t}dt \qquad (39)$$

Proof

To recover the future share prices changes for both Banks under study

we calculate the cumulative return f a stock over a period of time; by integrating

$$\text{From (38)} \quad F_{BANK} = 0.42e^{0.08t}dt$$

$$F_{BANK} = 0.42\frac{e^{0.08t}}{0.08} + K_1$$

similarly for Access bank

$$\text{From (39)} \quad A_{ACCESS} = 1.15e^{0.18t}dt$$

$$A_{ACCESS} = 1.15\frac{e^{0.18t}}{0.18} + K_2$$

Table 3: Analysis of future price changes through propostion 1 for Fidelity and Access Bank

Time (month)	Fidelity bank	**Access bank**	**Mean**	**STD**
0	5.5	6.6389	6.0694	0.8053
1	5.9373	7.8989	6.9181	1.3891
2	6.4109	9.4074	7.9092	2.1188
3	6.9241	11.2134	9.0687	3.0330
4	7.4799	13.3755	10.4277	4.1688
5	8.0821	15.9641	12.0231	5.5734
6	8.7344	19.06323	13.8988	7.3036
7	9.4410	22.7735	16.1073	9.4275
8	10.2065	27.2156	18.7111	12.027
9	11.2065	32.5336	21.7847	15.2013

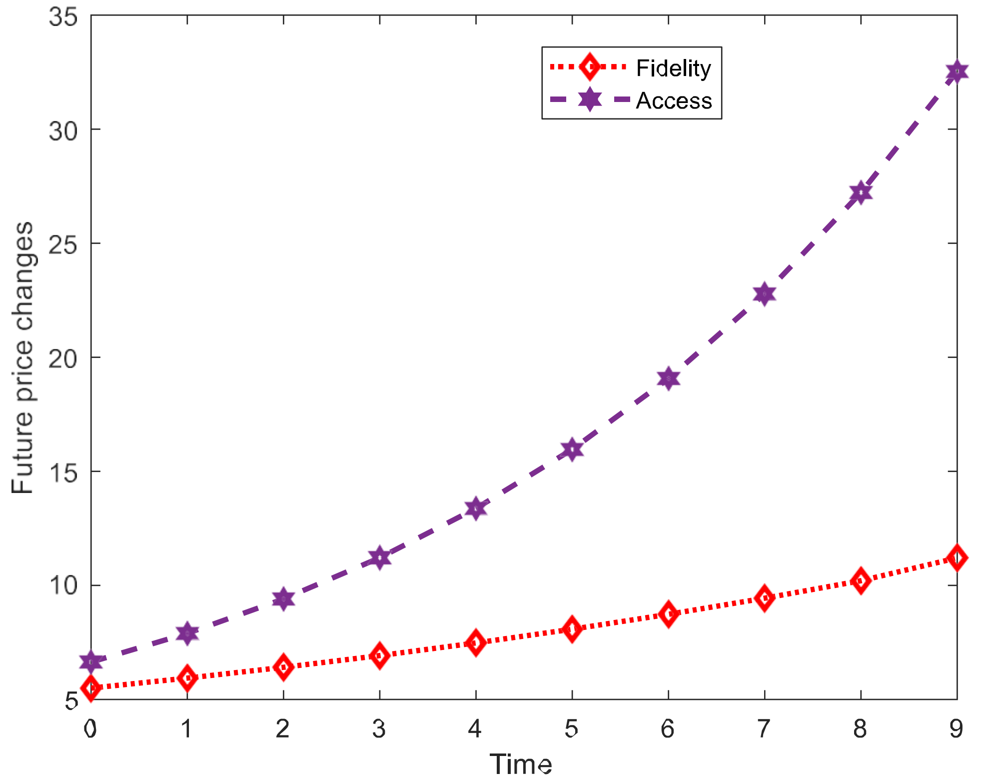

Figure 1: The profiles of Future price Changes over time

Table 4: The Effects of volatility via Proposition 1 on Future price changes with respect to time for Fidelity and Access Bank

Time (month)	Volatility	Fidelity bank	Access bank
0	0.25	5.45	6.5889
1	0.3	5.9373	7.8989
2	0.35	6.4609	9.4574
3	0.4	7.0241	11.3134
4	0.45	7.6299	13.5255
5	0.5	8.2821	16.1641
6	0.55	8.9844	19.3132
7	0.6	9.741	23.0735
8	0.65	10.5565	27.5656
9	0.7	11.4358	32.9336

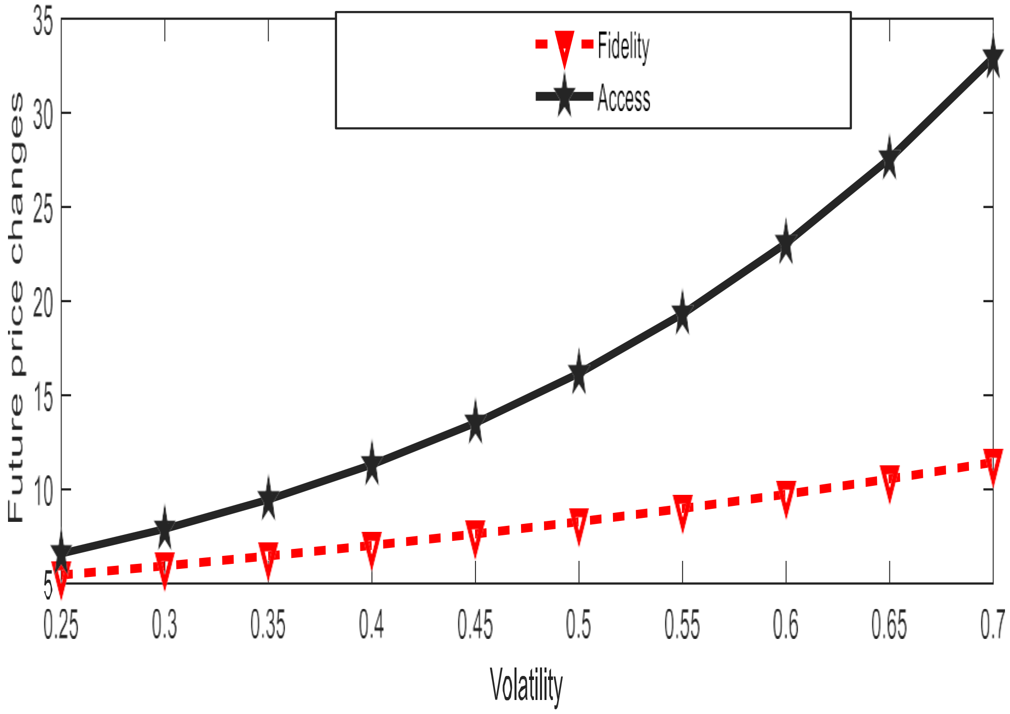

Figure 2: The Profiles of Future Price Changes against Volatility

5. Discussion of Results

Using (33) and (34) gives the Call option values columns 2 of Tables 1 and 2 respectively. The Call option prices with two different maturity dates show that investor or the banks are more flexible to adjust positions based on changing market conditions. The difference between two Call Option prices of different expiration dates can provide insight into market's expectations for the underlying security's future price movements. A larger difference between two Call option prices with different expiration dates generally indicates a greater degree of volatility in the price of the underlying security to be more volatile, and will generally price the option with longer expiration date higher reflecting the increased potentials for the Option to be in the money at expirations for Fidelity and Access banks.

However, in Section 4.2, H_1 was accepted for both Fidelity and Access and concluded

that there is significance differences between the variances of Call Option prices at different expiration dates. The two variances: 0.8954 and 0.9746 signifies the degree of variations in prices of the two call options. A larger variance indicates that the prices of Call options are more volatile, meaning that they are more likely to change in value over time. While smaller variance indicates that the prices of Call options are less volatile, meaning that they are less likely to change over time.

Figure 1 shows that future price changes is independent of time; which means as time continue to increase, prices will also increase throughout the trading days. In Figure 2 indicates that the higher volatility increases the value of future price changes. All this are informative to corporate bodies or investors during the period of trading.

The Proposition 1 is useful for investors who want to see how much the stock has increased or decreased in value over time.

Table 3 shows increase time increase the future price changes of share prices; this increase indicates the banks are performing well and investors have confidence in the banks' future prospects. This can be a positive sign for investors, as it means that the value of their investment is increasing. The mean share price can be comparing different stocks or sectors, which can be helpful for diversifying portfolios. Share price standard deviation is measure of volatility or the extent to which a data set varies from its mean value. In the context of Call Option with different maturity dates, standard deviation tells how much the price of the options is likely to fluctuate over time.

Higher volatility means that there are greater potentials for a stock's price to move up or down. This means that investors who buys shares in a volatile stock have the potential to make a higher return on their investment if the stock price increases. This high volatility also attracts more investors to a stock as they see it as an opportunity to make a quick profit. In all some investors see volatility as a sign of a healthy market and may be more willing, see Table 4.

5.2 Conclusion

This project studied the framework of Black-Scholes model of European Call option and Senedecor's F-distribution in pricing options on share price of Fidelity and Access banks which gave closed form prices of Call options with two different dates of expiration, The difference between two Call option prices of different expiration dates provides insight into market's expectations for the underlying security's future price movements. In hypothesis testing H1 were accepted for both Fidelity and Access and revealed that there is significance differences between the variances of Call Option prices at different expiration dates. However, a proposition was developed and proved; which were used in the analysis of future price changes for decision making. To this end, means of future prices as well as their respective standard deviations affecting real life changes for capital markets were considered which is informative in terms of investment returns.

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