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A Model of Investment Trend Functions: Principal Component Analysis

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Abstract

The applications of matrix method with stochastic terms were established in modeling share prices through quadratic and seasonal trend functions. In particular, the methods of rate of change were adopted to assess the value of share prices on two trend functions. From the results, Mean Squared Error (MSE) was used as a criterion for selection, based on this trend functions: the results show that the Seasonal trend function overtook Quadratic trend. More so, this study also applies Principal Component Analysis (PCA) to analyze the share prices of Access Bank, Fidelity and Merged Bank. The results show that the first principal component explains 81.5% of the variance in the data, indicating a high degree of correlation between the share prices of the three banks. The study provides insight into the relationships between the share prices of the three banks and identifies the underlying factors driving the variance in the data.

Keywords:

Principal Component Analysis, Mean Squared Error, Share prices, Matrix and Stochastic Analysis.

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1.1 Introduction

Financial market are complex systems influenced by numerous factors, making it challenging to identify underlying patterns and trend. Principal Component Analysis (PCA) offers a powerful tool for simplifying and understanding market dynamics by extracting key components that drive market behavior. Using a matrix approach, PCA transforms a set of orthogonal variables, called principal components, which capture the majority of the data's variance, see [1-4]. However, capital market involves the problems and prospect of equity investment. This involves the issue and market of shares, bonds and debentures using the services of brokers, dealers and underwriters. The capital market provides a means through which this is made possible. However, the paucity of long-term capital has posed the greatest. In capital investments monies are been invested in a business such that the return rates will be appropriately utilized to cover day-to-day trading activities expenses. An investor may need additional capital assets; in order to improve in its trading events. So capital investments are established on the basis of: to acquire additional capital assets for expansion which enables the business to increase in unit production, create new ideas on products or even add value to the business and to explore on technological advancements in order to increase efficiency and reduce costs and to replace worn-out assets. In absence of capital investments, trading business will definitely have a hard time getting off the ground.

Nevertheless, lots of authors on stock prices using different approaches in their respective modeling, for instance [5] considered the stochastic analyses of Markov chain in finite states which enabled them to proffer precise condition of obtaining expected mean rate of return of each stock. [6] Considered a matrix application to Dangote stock market prices is considered where an illustrative case is provided in different forms. In a similar manner, [7] examined stochastic system with changes to measure the value of wealth for each corporate investor through linear and quadratic returns. Also, [8] Investigated system of stochastic differential equations with prominence on disparities of drift parameter for stock market. In [9] the stochastic analysis of two asset values was successfully analyzed. [10]. Considered stability and controllability for stock exchange market were obtained; first by developing a vector valued stochastic differential system with control. [11] Investigated the applications of various stochastic volatility models in determining optimal investment strategies in the stock market. Yet [8] studied stochastic model of the fluctuation of stock market price. Numerous authors have comprehensively addressed the issues of stock prices namely [14 – 17].

This study applies the matrix approach and PCA to model market trend functions, aiming to uncover the underlying structure of market movements and provide insights for investors and analysts.

The paper is set as follows: Section 2.1 is Material and methods, Section 3.1 presents results and discussion while the paper is concluded in Section 4.1.

2.1 Material and Methods

Here, our focus in Section 3.1 stochastic differential equation, which gave rise to matrix algebra with stochastic terms.

2.1.1 Stochastic Differential Equations

Here, we consider a market where the underlying asset price v , $0 \le t \le T$ on a complete probability space $(\Omega, f, \wp)$ is governed by the following stochastic differential equation:

$$dS(t) = \alpha S(t)dt + \sigma dw(t).\ 0 < v < \infty\ .$$

2.2 Problem Formulation

We employ matrix approach to model the share prices of Access Bank, Fidelity Bank and Merged Bank. Let $A_i(t)$, $F_i(t)$ *and* $M_i(t)$ represents the share prices of these banks at time t ,where $i = 1,2,3$ corresponding to the respective trends. Precisely: $A_1(t), F_1(t)$, and $M_1(t)$ is a quadratic trend, $A_3(t), F_3(t)$, and $M_3(t)$ is a seasonal trend. Suppose the future investments plans, having a good knowledge of stochastic terms in (2.1) shows as follows:

- **Matrix of Quadratic trend function with stochastic terms:** A quadratic trend function with stochastic terms can be represented as follows:

$$A_1(t) = \begin{pmatrix} Xt^2{}_{011} + \sigma W_t & Xt^2{}_{012} + \sigma W_t & Xt^2{}_{013} + \sigma W_t \\ Xt^2{}_{021} + \sigma W_t & Xt^2{}_{022} + \sigma W_t & Xt^2{}_{023} + \sigma W \\ Xt^2{}_{031} + \sigma W & Xt^2{}_{032} + \sigma W & Xt^2{}_{033} + \sigma W_t \end{pmatrix} \quad (2.2)$$

$$F_1(t) = \begin{pmatrix} Xt^2{}_{011} + \sigma W_t & Xt^2{}_{012} + \sigma W_t & Xt^2{}_{013} + \sigma W_t \\ Xt^2{}_{021} + \sigma W_t & Xt^2{}_{022} + \sigma W_t & Xt^2{}_{023} + \sigma W \\ Xt^2{}_{031} + \sigma W & Xt^2{}_{032} + \sigma W & Xt^2{}_{033} + \sigma W_t \end{pmatrix} \quad (2.3)$$

$$M_1(t) = \begin{pmatrix} Xt^2{}_{011} + \sigma W_t & Xt^2{}_{012} + \sigma W_t & Xt^2{}_{013} + \sigma W_t \\ Xt^2{}_{021} + \sigma W_t & Xt^2{}_{022} + \sigma W_t & Xt^2{}_{023} + \sigma W \\ Xt^2{}_{031} + \sigma W & Xt^2{}_{032} + \sigma W & Xt^2{}_{033} + \sigma W_t \end{pmatrix} \quad (2.4)$$

- **Matrix of Seasonal trend Function with stochastic terms:** A matrix of cubic trend function with stochastic terms can be represented as follows:

$$A_3\left(t\right)=\begin{pmatrix} XSin(t)_{011}+\sigma W_t & XCos(t)_{012}+\sigma W_t & XSin(t)_{013}+\sigma W_t \\ XSin(t)_{021}+\sigma W_t & XCos(t)_{022}+\sigma W_t & XSin(t)_{023}+\sigma W_t \\ XSin(t)_{031}+\sigma W_t & XCos(t)_{032}+\sigma W_t & XSin(t)_{033}+\sigma W_t \end{pmatrix} \quad (2.5)$$

$$F_3\left(t\right)=\begin{pmatrix} XSin(t)_{011}+\sigma W_t & XCos(t)_{012}+\sigma W_t & XSin(t)_{013}+\sigma W_t \\ XSin(t)_{021}+\sigma W_t & XCos(t)_{022}+\sigma W_t & XSin(t)_{023}+\sigma W_t \\ XSin(t)_{031}+\sigma W_t & XCos(t)_{032}+\sigma W_t & XSin(t)_{033}+\sigma W_t \end{pmatrix} \quad (2.6)$$

$$M_3\left(t\right)=\begin{pmatrix} XSin(t)_{011}+\sigma W_t & XCos(t)_{012}+\sigma W_t & XSin(t)_{013}+\sigma W_t \\ XSin(t)_{021}+\sigma W_t & XCos(t)_{022}+\sigma W_t & XSin(t)_{023}+\sigma W_t \\ XSin(t)_{031}+\sigma W_t & XCos(t)_{032}+\sigma W_t & XSin(t)_{033}+\sigma W_t \end{pmatrix} \quad (2.7)$$

where $X_{011}+,\ldots,+X_{033}$ represents initial monthly share prices under different trend functions at time t ,σ is the volatility of the underlying share prices of the three banks under study, W_t is Brownian motion or wieners process and Sin, Cos represent seasonal parameters according to [18].

2.2.1 Method of Solution

This Section, (2.2-2.4) we solve independently using the method of rate of change. Therefore, solving from (2.2-2.4) which is quadratic trend function with stochastic terms gives the following:

$$\frac{dA_1\left(t\right)}{dt}=\begin{pmatrix} 2Xt_{011}+\sigma\frac{W_t}{dt} & 2Xt_{012}+\sigma\frac{W_t}{dt} & 2Xt_{013}+\sigma\frac{W_t}{dt} \\ 2Xt_{021}+\sigma\frac{W_t}{dt} & 2Xt_{022}+\sigma\frac{W_t}{dt} & 2Xt_{023}+\sigma\frac{W_t}{dt} \\ 2Xt_{031}+\sigma\frac{W_t}{dt} & 2Xt_{032}+\sigma\frac{W_t}{dt} & 2Xt_{033}+\sigma\frac{W_t}{dt} \end{pmatrix} \quad (2.8)$$

$$\frac{dF_1\left(t\right)}{dt}=\begin{pmatrix} 2Xt_{011}+\sigma\frac{W_t}{dt} & 2Xt_{012}+\sigma\frac{W_t}{dt} & 2Xt_{013}+\sigma\frac{W_t}{dt} \\ 2Xt_{021}+\sigma\frac{W_t}{dt} & 2Xt_{022}+\sigma\frac{W_t}{dt} & 2Xt_{023}+\sigma\frac{W_t}{dt} \\ 2Xt_{031}+\sigma\frac{W_t}{dt} & 2Xt_{032}+\sigma\frac{W_t}{dt} & 2Xt_{033}+\sigma\frac{W_t}{dt} \end{pmatrix} \quad (2.9)$$

$$\frac{dM_1(t)}{dt}=\begin{pmatrix} 2Xt_{011}+\sigma\frac{W_t}{dt} & 2Xt_{012}+\sigma\frac{W_t}{dt} & 2Xt_{013}+\sigma\frac{W_t}{dt} \\ 2Xt_{021}+\sigma\frac{W_t}{dt} & 2Xt_{022}+\sigma\frac{W_t}{dt} & 2Xt_{023}+\sigma\frac{W_t}{dt} \\ 2Xt_{031}+\sigma\frac{W_t}{dt} & 2Xt_{032}+\sigma\frac{W_t}{dt} & 2Xt_{033}+\sigma\frac{W_t}{dt} \end{pmatrix} \quad (2.10)$$

The derivatives of (2.5-2.7) gives the following share price matrices:

$$\frac{dA_3(t)}{dt}=\begin{pmatrix} XCos(t)_{011}+\sigma\frac{dW_t}{dt} & -XSin(t)_{012}+\sigma\frac{dW_t}{dt} & XCos(t)_{013}+\sigma\frac{dW_t}{dt} \\ XCos(t)_{021}+\sigma\frac{dW_t}{dt} & -XSin(t)_{022}+\sigma\frac{dW_t}{dt} & XCos(t)_{023}+\sigma\frac{dW_t}{dt} \\ XCos(t)_{031}+\sigma\frac{dW_t}{dt} & -XSin(t)_{032}+\sigma\frac{dW_t}{dt} & XCos(t)_{033}+\sigma\frac{dW_t}{dt} \end{pmatrix} \quad (2.11)$$

$$\frac{dF_3(t)}{dt}=\begin{pmatrix} XCos(t)_{011}+\sigma\frac{dW_t}{dt} & -XSin(t)_{012}+\sigma\frac{dW_t}{dt} & XCos(t)_{013}+\sigma\frac{dW_t}{dt} \\ XCos(t)_{021}+\sigma\frac{dW_t}{dt} & -XSin(t)_{022}+\sigma\frac{dW_t}{dt} & XCos(t)_{023}+\sigma\frac{dW_t}{dt} \\ XCos(t)_{031}+\sigma\frac{dW_t}{dt} & -XSin(t)_{032}+\sigma\frac{dW_t}{dt} & XCos(t)_{033}+\sigma\frac{dW_t}{dt} \end{pmatrix} \quad (2.12)$$

$$\frac{dM_3(t)}{dt}=\begin{pmatrix} XCos(t)_{011}+\sigma\frac{dW_t}{dt} & -XSin(t)_{012}+\sigma\frac{dW_t}{dt} & XCos(t)_{013}+\sigma\frac{dW_t}{dt} \\ XCos(t)_{021}+\sigma\frac{dW_t}{dt} & -XSin(t)_{022}+\sigma\frac{dW_t}{dt} & XCos(t)_{023}+\sigma\frac{dW_t}{dt} \\ XCos(t)_{031}+\sigma\frac{dW_t}{dt} & -XSin(t)_{032}+\sigma\frac{dW_t}{dt} & XCos(t)_{033}+\sigma\frac{dW_t}{dt} \end{pmatrix} \quad (2.13)$$

2.2.3 Comparison of Estimation on the Trend functions

We have earlier defined our trend functions such as: Quadratic trend, and Seasonal trend. The Mean Square Errors (MSE) shall be used as a criterion for selection of the best trend for each of banks.

$$MSE=\left(\frac{1}{n}\right)\Sigma\left(y_i-\hat{y}_t\right)^2 \quad (2.14)$$

where y_i is the actual share prices of Access Bank, Fidelity Bank and Merged Bank, $\hat{y}_i$ is the predicted values, n is the number of data points and $\sum$ is summation symbols , indicating the sum of the values. The method with the minimum mean squared error (MMMSE) becomes the best methods for the estimation of trend functions.

2.3 Principal Component Analysis (PCA) OF Shares Prices.

Let $A_1, F_1,$ and M_1 represents the share prices of Access Bank, Fidelity Bank and Merged Bank.

Data Matrix: $X = [A_1, F_1, M_1], [A_2, F_2, M_2] \cdots [A_n, F_n, M_n]$ where n is the number of observations.

Standardization:

$$\left.\begin{aligned} A\_Std &= \left(A - \mu\_A\right) / \sigma\_A \\ F\_Std &= \left(F - \mu\_F\right) / \sigma\_F \\ M\_Std &= \left(M - \mu\_M\right) / \sigma\_M \end{aligned}\right\} \qquad (2.15)$$

where μ_A, μ_F *and* μ_M are the means and σ_A, σ_F *and* σ_M are the standard deviations of A, F *and* M.

Covariance Matrix:

$$\begin{aligned} \Sigma = &| \operatorname{var}\left(A\_Std\right), Cov\left(A\_Std, F\_Std\right), Cov\left(A\_Std, M\_Std\right) | \\ &| \operatorname{Cov}\left(F\_Std, A\_Std\right), \operatorname{Var}\left(F\_Std\right), Cov\left(M\_Std, \mathrm{F}\_Std\right) Var\left(M\_Std\right) | \end{aligned} \qquad (2.16)$$

Eigenvalue Decomposition:

$$\Sigma = V \Lambda V^T \qquad (2.17)$$

Where, V is the matrix of eigenvalues, Λ is the diagonal matrix of eigenvalues and T denotes the transpose.

Principal Components:

$$PC1 = V_{11} - A\_Std + V_{12} - F\_Std + V_{13} - M\_Std \quad (2.18)$$

$$PC2 = V_{21} - A\_Std + V_{22} - F\_Std + V_{23} - M\_Std \quad (2.19)$$

$$PC3 = V_{31} - A\_Std + V_{32} - F\_Std + V_{33} - M\_Std \quad (2.20)$$

where $V_{11}, V_{12} + \cdots + V_{33}$ are the coefficient of the eigen vectors.

Eigenvalues and Eigenvectors: The $\lambda_1, \lambda_2, \lambda_3$ represents the eigenvalues of $\sum$, V_1, V_2, V_3 represents eigenvectors of $\sum$.

$$\left.\begin{aligned} S1 &= X * V_1 \\ S2 &= X * V_2 \\ S3 &= X * V_3 \end{aligned}\right\} \qquad (2.21)$$

where $S1, S2$ *and* $S3$ are the scores of the principal components, the idea behind the above concepts can be seen [1-4].

3.1 Results and Discussion

This Section presents analyzed results whose methods are stated in Section 2.1. Hence, we have the following parameter values:
$\text{n} = 0.2,\ 0.5\ , 0.8,\ \text{dW} = 0.002,\ \sigma = 0.25,\ dt = 0.005$ and $t = 1$. which were implemented using Matlab programming software:

Analysis of Rate of Change on the Share prices according to their Trend Functions

$$\frac{dA_1(t)}{dt} = \begin{pmatrix} 820.1 & 160.1 & 252.1 \\ 158.1 & 196.1 & 184.1 \\ 254.1 & 182.1 & 756.1 \end{pmatrix}, \frac{dF_1(t)}{dt} = \begin{pmatrix} 830.1 & 124.1 & 270.1 \\ 122.1 & 242.1 & 162.1 \\ 278.1 & 160.1 & 568.1 \end{pmatrix}$$

$$\frac{dM_1(t)}{dt} = \begin{pmatrix} 1650.1 & 284.1 & 528.1 \\ 280.1 & 438.1 & 162.1 \\ 532.1 & 160.1 & 568.1 \end{pmatrix}$$

This trend function captures non-linear relationship between variables. The results suggest that the quadratic trend has a significant impact on share prices, indicating that the rate of change in share prices is influenced by the quadratic term.

$$\frac{dA_3(t)}{dt} = \begin{pmatrix} 409.631 & -1.496 & 125.9866 \\ 79.0289 & -1.7101 & 92.0172 \\ 126.9857 & -1.58795 & 377.7598 \end{pmatrix}, \frac{dF_3(t)}{dt} = \begin{pmatrix} 414.7265 & -1.1819 & 137.9758 \\ 61.0451 & -2.21145 & 81.0271 \\ 138.9749 & -1.496 & 283.8444 \end{pmatrix}$$

$$\frac{dM_3(t)}{dt} = \begin{pmatrix} 824.3575 & -2.6779 & 263.8624 \\ 139.974 & -3.92155 & 172.9443 \\ 265.8606 & -3.08395 & 761.4142 \end{pmatrix}$$

This trend function captures periodic fluctuations in share prices. The results suggest that the seasonal trend has a significant impact on share prices, indicating that share prices are influenced by seasonal factors. The negative predicted share prices indicate potential crashes or significant decline in share prices during the period of trading.

Table 3.1: Summary of Best Trend Function for Trading of Shares

QUADRATIC	TREND	FUNCTION	
Banks	**Access Bank**	**Fidelity**	**Merged Bank**
MSE	9.2346e+05	2.1218e+05	2.9917e+05
SEASONAL	**TREND**	**FUNCTION**	
MSE	8.3662e+03*	7.9983e+03*	1.1578e+05*

Table 3.1 talks about the comparison of trend function using Mean Squared Error (MSE). The results show that the seasonal trend function has the least minimal value of MSE, indicating it is the best performed among the two trend functions. This suggests that the seasonal that the seasonal trend function is better suited for modeling the share prices of the three banks. These results has some implications for share prices of access Bank, Fidelity Bank and Merged Bank. The seasonal trend function can be used to predict share prices and identify potential opportunities for investors. The Quadratic and Cubic trend functions may not be suitable for predicting share prices, but can still provide insights into the complex relationships between interest rates and share prices.

Principal Component Analysis of Shares Prices

$$\Sigma = |14491.11 \ 1181.39 \ 2411.11|$$
$$|1181.39 \ 15421.11 \ 2495.83|$$
$$|2411.11 \ 2495.83 \ 59321.11|$$
$$\lambda_1 = 62821.41,\ \lambda_2 = 14391.31, \lambda_3 = 0.00$$

$$V_1 = [0.33, 0.34, 0.88],\ \mathrm{V}_2 = [-0.83, 0.54, 0.13], V_3 = [0.45, 0.77, -0.45]$$

Principal Components:

$$PC1 = 0.33 - A\_Std + 0.34 - F\_Std + 0.88 - M\_Std$$

$$PC2 = -0.83 - A\_Std + 0.54 - F\_Std + 0.13 - M\_Std$$

$$PC3 = 0.45 - A\_Std + 0.77 - F\_Std - 0.45 - M\_Std$$

Scores

$PC1$	$PC2$ and	$PC3$
11041.19	−132.19	10.51
2226.19	−46.19	−1.51
3415.19	−64.19	2.51
4222.19	−40.19	−0.51
5343	−58.19	1.51
6276.19	−44.19	0.51
7417.19	−66.19	2.51
8272.19	−42.19	0.51
91121.19	−142.19	10.51

The first principal component (PC1) explains 81.5% of the variance in the data. The second principal component (PC2) explains 18.5% of the variance in the data. The third principal component (PC2) explains 18.5% of the variance in the data. The result suggests that the share prices of the three banks are highly correlated, with the first principal component explaining most of the variance in the data. The second principal component explains some of the remaining variance, indicating that there are some differences between the share prices of the three banks.

4.1 Conclusion

The study concludes that the seasonal trend function is most suitable for modeling share prices of Access Bank, Fidelity and Merged Bank. The results show that the seasonal trend of function provides a good fit to the data, while the Quadratic perform poorly. The study's findings have implications for investors, financial analysts, and policy makers, and suggest that the seasonal trend function can be used to inform investment decision and risk management strategies. The study also concludes that the share prices of Access Bank, Fidelity Bank and Merged Bank are highly correlated, with the first principal component explaining the majority of the variance in the data. The results of the study have implications for investors and financial analysis seeking to understand the relationships between the share prices of different bank

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