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ON THE STRUCTURE OF TWO - FOLD FUZZY SET

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Abstract

The traditional fuzzy set deals with single degree of membership which cannot capture matters exhibited by multi-dimensional uncertainty. This gap motivate is the motivation to our work. We extend the existing Boolean algebra of the traditional fuzzy set to the two-fold fuzzy set., to further make its application wider and easier. The results show the conformity of the two-fold fuzzy sets with the existing Boolean algebra in the theory of the traditional fuzzy sets. The basic operations of the traditional fuzzy set were investigated against the two-fold fuzzy sets. The axioms of fuzzy union, fuzzy intersection and fuzzy complement of the traditional fuzzy set were investigated and extended to that of the two-fold fuzzy set. Furthermore, the set having satisfied the axioms was proved to be commutative, associative, distributive, involutivity, absorptive point-wise, and in concord with De Morgan's law. With this the two – fold fuzzy set will be more relevant in modelling vagueness and imprecision than the traditional fuzzy set.

Keywords:

Fuzzy operators, fuzzy set, membership, two-fold fuzzy set, uncertainty.

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1 Introduction

The concept of fuzzy set was introduced by Zadeh (1965) where he defined the set in terms of mapping from a set into the unit interval on the real line. This provided a means of mathematically describing situations which give rise to ill – define or collections of objects for which there is no precise criteria for membership. In a collection of such type of vague or fuzzy boundaries there are objects for which it is impossible to determine whether or not they belong to the collection Zadeh (1978). In real situations such as pattern classification, fuzziness is the rule rather than the exception. Fuzzy sets can be applied to such problems than other methods being used according to P. V. S Reddy (2020).

The extension of the traditional fuzzy set to two - fold fuzzy set was proposed by Dedier and Preade (1987). Here, each element has dual representation: belief and disbelief functions. Their work paved way for several researches in the twenty first century. Consequently, the concept of fuzzy set theory has proved to be a powerful tool for modelling and dealing with uncertainties in various domains providing a single degree of membership for each element, allowing for gradual membership rather than strict distinctions.

However, in real life uncertainty exhibits complex patterns that cannot be captured by a single degree of membership. This limitation has been addressed by the two – fold fuzzy set which enhances flexibility in representing complex and multi – dimensional uncertainty.

In our work, we investigate the theoretical foundation and some of the properties of the two – fold fuzzy sets and extend the traditional fuzzy set properties to the two – fold fuzzy set. This makes its application wider and easier.

2 Definition of Basic Terms

In this section we define basic concepts, state and or prove theorems and propositions related to our work based on P. V. S. Reddy (2020), Zadeh, L. A (1978), Alhaji, J. et al (2021) and Zimmermann, H. J (1996).

2.1. Definition

If X is a collection of objects denoted generically by x, then $\hat{A}$ is a fuzzy set in X defined by an ordered pair: $\hat{A} = \{(x, \mu_{\hat{A}}(x)) : x \in X\}$, Where; $\mu_{\hat{A}}(x)$ is called a membership function (generalized characteristic function) which maps X to the membership space.

2.2 Remark

The membership function here is crisp (real valued) function. For simplicity, we shall be using A, B or C without cap to represent fuzzy sets A, B and C respectively.

2.3. Definition

If X is a collection of objects denoted generically by x, then a fuzzy set A in X is;

$$\mu_A(x) = \begin{cases} 1, if\ x\ is\ completely\ in\ A \\ 0, if\ x\ is\ completely\ not\ in\ A \\ (0,1), if\ x\ is\ partly\ in\ A \end{cases}$$

Where $\mu_A(x)$ is the membership function of x in A, 1 represents full membership and 0 represents full non-membership of x in A. Equivalently, we have the next definition.

2.4. Definition

A fuzzy set A in a universe of discourse X is defined as its membership function μ_A (x):→ [0,1], x ∈ X. where $A= \frac{\mu_A(x_1)}{x_1} + \frac{\mu_A(x_2)}{x_2} + \frac{\mu_A(x_3)}{x_3} ...+ \frac{\mu_A(x_n)}{x_n}$, "+" is union
Therefore, a finite fuzzy set has the general form:
$A = \sum_{x \in X} \mu_A(x)/x$
Accordingly, we have:

2.5. Definition

Let A and B be to two fuzzy sets in a universe of discourse X defined by
$A = \{(x, \mu_A(x)): x \in X\}$
$B = \{(x, \mu_B(x)): x \in X\}$
Next, we define the union and intersection of fuzzy sets analogous to union and intersection of ordinary sets.

2.6 Definition

The Union and Intersection of A and B are defined with respect to their membership functions as; $\mu_{A \cup B}$ (x) = max $\{(\mu_A(x), \mu_B(x)): x \in X\}$, and $\mu_{A \cap B}$ (x) = min $\{(\mu_A(x), \mu_B(x)): x \in X\}$, respectively.

2.7 Remark

Let A be a nonempty set (cantorian) in a universe X. then, A can be described or characterized via a function called the characteristics or membership or discrimination function μ_A: X→ [0,1] defined as:

$$\mu_A(x) = \begin{cases} 1, & if\ x \in A \\ 0, & otherwise \end{cases}$$

Complementation in fuzzy sets is defined in 2.8 with respect to membership function.

2.8 Definition

If A is a fuzzy set in X, then the complement (Negation) of A denoted by A^c is defined with respect to the membership function as; $A^c = \mu_{A'}(x) = \{(x, 1-\mu_A(x)): x \in X\}$
What follows is the algebra of the traditional fuzzy set

2.9. Definition

Let A, B, and C be three fuzzy sets in X, the following results hold;

a- $A \cup B = B \cup A$ and $A \cap B = B \cap A$ (Commutativity)
b- $(A \cup B) \cup C = A \cup (B \cup C)$ and $(A \cap B \cap C = A \cap (B \cap C)$ (Associatvity)
c- $(A \cup B \cap C = (A \cap C) \cup (A \cap C)$ (Disributivity)
$(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$
d- If $A \leq B$, then $A \cup B = B$ and $A \cap B = A$ (Absorption law)

2.10 Remark

The definitions that follow are due to Didier and Henri (1987) and Dubois et al (1988) and later studied by P. V. S Reddy (2017, 2013 & 2020) and Mouzon et al (2021)

2.11 Definition

A two-fold fuzzy set A in a universe of discourse X is defined by its membership function;

μ_A (x) → [0,1]. Where $A = \{ \mu_A^{belief}$(x), $\mu_A^{disbelief}$(x)} with x ∈ X. and

μ_A^{belief}(x) and $\mu_A^{disbelief}$(x) are fuzzy membership functions of the two-fold fuzzy set.

$\mu_A^{belief}(x) = \mu_A^{belief}(x_1)/x_1 + \mu_A^{belief}(x_2)/x_2 + \mu_A^{belief}(x_3)/x_3 + \ldots + \mu_A^{belief}(x_n)/x_n$

And;

$\mu_A^{disbelief}(x) = \mu_A^{disbelief}(x_1)/x_1 + \mu_A^{disbelief}(x_2)/x_2 + \mu_A^{disbelief}(x_3)/x_3 + \ldots +$ $\mu_A^{disbelief}(x_n)/x_n$ where "+" is union, with the following restrictions and interpretations.

$\mu_A^{belief}(x) + \mu_A^{disbelief}(x) > 1$, implies redundant information,

$\mu_A^{belief}(x) + \mu_A^{disbelief}(x) < 1$, implies insufficient information, and

$\mu_A^{belief}(x) + \mu_A^{disbelief}(x) = 1$, which implies sufficient information

2.12 Remark

We shall use μ_A (x) and v_A (x) to represent the belief and disbelief functions in the rest of our work

From definition 2.6 we have:

2.13 Definition

Let A and B be two-fold fuzzy sets, the union and intersection of A and B are defined with respect to their membership functions using max-min operators as follows

$\mu_{A \cup B}$ (x) = {max (μ_A x), μ_B (x)), max ($v_{\hat{A}}$ (x), v_B (x))}

$\mu_{A\cap B}$ (x) = {min ($\mu_{\hat{A}}$ (x), μ_B(x)), min (v_A (x) v_B (x))}

2.14. Definition

Let A be a two-fold fuzzy set, then complement or negation of A is defined with respect to its membership function as; μ_{A^c} (x) = {(1 – μ_A (x), (1 – v_A (x))}

2.15 Remark

Fuzzy sets theory is fundamentally built upon set theory operations which are basically; union, Intersection and Complementation. However, the operations in fuzzy set and its extensions such as Intuitionistic, type 2, Two-fold fuzzy sets, etc, are achieved using max-min operation. Thus for any fuzzy set extension, the definitions and propositions that follow are major tools in determining our results.

2.16 Definition

Let A, B, C and D ∈ F(X) be type 1 (traditional) fuzzy sets (T1FS) . Then the standard operators on T1FS are as follows

1. μ_{A^c} (x) =1- μ_A (x) the complement operator,
2. $\mu_{A \cup B}$ (x) = max {(μ_A (x), μ_B (x)): x∈ X}, the joint operator (or union)

3. $\mu_{A\cap B}$ (x) = min {(μ_A (x), μ_B (x)):x∈ X}, the meet operator (or intersection)

These are the basic operators upon which the non-standard operators will be derived.

2.17 Proposition

Considering the basic connectives in fuzzy set theory, the following properties hold in fuzzy set.

1. $(A^C)^C$ = A. Involution
2. A∪B= B∪A and A∩B = B∩A Commutativity
3. (A∪B)∪C = A∪(B∪C) and (A∩B)∩C = A∩(B∩C) Associativity
4. A∩(B∪C) = (A∩B) ∪ (A∩C) and A∪ (B∩C) =(A∪B) ∩ (A∪C) Distributivity
5. A∩A = A and A∪A = A Idempotency
6. A∪(A∩B) = A and A∩(A∪B) = A Absorption by A
7. A∪X = X and A∩ ∅ = ∅ Absorption by X and ∅ respectively
8. A ∪ ∅ = A and A ∩ X = A Identity
9. $\overline{A \cap B} = \bar{A} \cup \bar{B}$ De Morgan's

We state the next proposition without prove.

2.18 Proposition

If A is a non-classical fuzzy set A: X→ [0,1].then
i- A ∩ $A^C \neq \emptyset$
ii- A ∪ $A^C \neq$ X

2.19 Definition

Let A, B and C be fuzzy sets. The fuzzy union, U:[0, 1]→ [0, 1]x[0, 1] satisfies the following.
1. U(0, 0) = 0, U(1, 0) =1, U(0, 1) =1, u(1, 1) =1. Boundary condition
2. If $\mu_A(x) < \mu_{A^c}(x)$ and $\mu_B(x) < \mu_{B^c}(x)$, then U($\mu_A(x)$ $\mu_B(x)$) ≤ U($\mu_{A^c}(x)$ $\mu_{B^c}(x)$) Monotonicity
3. U is commutative: U($\mu_A(x)$, $\mu_{\mathbf{B}}(x)$) = U($\mu_B(x)$, $\mu_A(x)$)
4. U is associative: U(U($\mu_A(x)\mu_B(x)$) $\mu_C(x)$) = U($\mu_A(x)$, U($\mu_B(x)\mu_C(x)$))
5. U is a continuous function
6. U($\mu_A(x)$, $\mu_A(x)$) = $\mu_A(x)$, Idempotency.

2.20 Definition

Let A, B and C be fuzzy sets, the fuzzy intersection, I:[0,1]x[0,1]→[0, 1]
Satisfies the axioms
1. I(0,0) = 0, I(1,0) = 0, I(0, 1) = 0, I(1,1) = 1 Boundary condition
2. If $\mu_A(x) < \mu_{A^c}(x)$ and $\mu_B(x) < \mu_{B^c}(x)$, then I($\mu_A(x)$, $\mu_B(x)$) ≤ I($\mu_{A^c}(x)$, $\mu_{B^c}(x)$)
3. Commutativity: I($\mu_A(x)$, $\mu_B(x)$) = I($\mu_B(x)\mu_A(x)$)
4. Associativity: I(I($\mu_A(x)\mu_B(x)$) $\mu_C(x)$) = I($\mu_A(x)$, I($\mu_B(x)$, $\mu_C(x)$))
5. I is a continuous function
6. I($\mu_A(x)\mu_A(x)$) = $\mu_A(x)$ Idempotency

2.21 Definition

Let A and B be fuzzy sets. A fuzzy complement written as C[0, 1]→[0, 1] satisfies the following axioms
1. $C(0) = 1, C(1) = 0$Boundary Condition
2. If $\mu_B(x) < \mu_A(x)$, then $\mu_{B^C}(x) \geq \mu_{A^C}(x)$
3. C is continuous
4. $C(C(A)) = A$

2.22 Remark

The boundary conditions in all the three set of axioms are clear and obvious by the max-min definition of union and intersection. Axioms 1-4 of definition 2.19 form axiomatic skeleton for fuzzy union, axioms 1-4 of 2.20 form axiomatic skeleton for fuzzy intersection while axioms 1 and 2 of 2.21 form the axiomatic skeleton for fuzzy complement. We illustrate these concepts in the next example.

2.23 Example

Let A and B be two-fold fuzzy sets in X. such that;
A= {(1, 0.5, 0.3), (2, 0. 4, 0.4), (3, 0.4, 0.3)} and;
B = {(1, 0.4, 0.3), (2, 0.3, 0.2), (3, 0.3, 0.1)}
Clearly, B < A and;
1. $A^C = \{(1, 0.5, 0.7), (2, 0.6, 0.6),(3, 0.6, 0.7)\}$, clearly $A < A^C$ and
$B^C = \{(1, 0.6, 0.7),(2, 0.7, 0.8),(3, 0.7, 0.9)\}$; also $B < B^C$
Now, A∪B = {(1, 0.5, 0.3),(2, 0..04, 0.4), (3, 0.4, 0.3)}
$A^C \cup B^C = \{(1, 0.6, 0.7), (2, 0.7, 0.8), (3, 0.7, 0.9)\}$
$\Rightarrow (A\cup B) \cup (A^C \cup B^C) = A^C \cup B^C$ and $(A\cup B) \cap (A^C \cup B^C) = (A\cup B)$
Thus, $A\cup B < (A^C \cup B^C)$

2. A∩B = {(1, 0.4, 0.3), (2, 0.3, 0.2),(3, 0.3, 0.1)} and

$A^C \cap B^C = \{(1, 0.5, 0.7), (2, 0.6, 0.6), (3, 0.6, 0.7)\}$

Clearly, $A\cap B < A^C \cap B^C$, Since $(A\cap B) \cup (A^C \cap B^C) = A^C \cap B^C$ and $(A\cap B) \cap (A^C \cap B^C) = A\cap B$

3. $A\cup B \leq A \Rightarrow B < A$ and we can deduce that $B^C \geq A^C$
Thus, from definition 2.19 (2), we have that monotonicity holds in all axioms.

3. Results and Discussion

3.1 Proposition

Let A, B and C be two-fold fuzzy sets, such that A>B>C and let

$A_x = \{\mu_A(x), v_A(x)\}, B_x = \{\mu_B(x), v_B(x)\}$, and $C_x = \{\mu_C(x), v_C(x)\}$. Then A, B and C satisfy the following properties:

1. Complement of $(A^C)^C = A$. — Involution
2. $A\cup B = B\cup A$ and $A\cap B = B\cap A$ — Commutativity
3. $(A\cup B)\cup C = A\cup(B\cup C)$ and $(A\cap B)\cap C = A\cap(B\cap C)$ — Associativity
4. $A\cap A = A$ and $A\cup A = A$ — Idempotency

5. A∪(A∩B) = A and A∩(A∪B) = A Absorption by A
6. A∪X = X and A ∩ ∅ = ∅ X and ∅ Absorption respectively
7. A ∪ ∅ = A and A ∩ X = A Identity
8. i. A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C)
   ii. A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C) Distributivity
9. $(A \cup B)^C = A^C \cap B^C$ and $(A \cap B)^C = A^C \cup B^C$ De Morgan's

Proof:

Let $A_x > B_x > C_x$ (Given)

1. Let A = {x, A_x}, Then,

$(A^C) = \{x, 1 - A_x\}$
$\Rightarrow (A^C)^C = \{x, 1- (1 - A_x)\} = \{x, A_x\}$
$\Rightarrow (A^C)^C = A_x$

2. i. A∪B = B∪A ii. A∩B = B∩A

i. consider LHS: A∪B= max(A_x, B_x) = A_x
from the RHS: B∪A= max(B_x, A_x) = A∪B= A_x
ii. The method is the same as i.

3. i. (A∪B)∪C = A∪(B∪C) ii. (A∩B)∩C = A∩(B∩C)

i. LHS: (A∪B)∪C = max(max(A_x, B_x), C_x) = max(= A_x, C_x) = A_x
RHS: A∪(B∪C) = max(A_x, max(B_x, C_x)) = max(= A_x, B_x) = A_x
ii. similar to i. using min-operator

4. i. A∪A =A. ii. . A∩A = A

i. If A = A_x then, A∪A = max(A_x, A_x) = A_x
ii. . A∩A min(A_x, A_x) = A_x

5. i. A∪(A∩B) = A. ii. A∩(A∪B) = A

i. . A∪(A∩B) = max(A_x, min(A_x, B_x)) = max(A_x, B_x) = A_x ⇒ A∪(A∩B) = A
ii. A∩(A∪B) = min(A_x, max(A_x, B_x)) = min(A_x, B_x) = A_x ⇒ A∩(A∪B) = A

6. i. A∪X = X ii. A ∩ ∅ = ∅

i. A∪X = X, since A is a two-fold fuzzy set in X then, it can at most expressed as A ⊆ X
⇒ max{A_x,X} = X.
ii. . A ∩ ∅ = min(A_x, 0) = 0 ⇒ A ∩ ∅ = ∅

7. i. A ∪ ∅ = A. ii. A ∩ X = A

i. A ∪ ∅ = max(A_x, 0) = A_x ⇒ A ∪ ∅ A
ii. A ∩ X = min(A, X) = A. since A is in X

8. i. A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C)

ii. A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C)

Now from the LHS of (i);

A ∪ (B ∩ C) = max[A_x, min(B_x, C_x)] = max(A_x, C_x) = A_x

and the RHS of (i):

(A ∪ B) ∩ (A ∪ C) = min[max(A_x, B_x), max(A_x, C_x)] = min[A_x , A_x] = A_x
⇒ LHS = RHS = A_x
ii. A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C)

Now, consider the LHS:
$A \cap (B \cup C) = \min[A_x, \max(B_x, C_x)] = \min[A_x, B_x] = B_x$
and the RHS:

$(A \cup B) \cap (A \cup C) = \max[\min(A_x, B_x), \min(A_x, C_x)] = \max[B_x, C_x] = B_x$

9. i. $(A \cup B)^C = A^C \cap B^C$ ii. $(A \cap B)^C = A^C \cup B^C$
i. $(A \cup B)^C = A^C \cap B^C$
LHS: $(A \cup B)^C = [(1 - \max(A_x, B_x)] = 1 - A_x$
RHS: $A^C \cap B^C = \min[(1 - A_x, 1 - B_x)] = 1 - A_x$ since $A_x > B_x \Rightarrow (1 - A_x) < (1 - B_x)$
$\Rightarrow (A \cup B)^C = A^C \cap B^C$ same apply to (ii) and the result hold.

3.2 Proposition

If A is a two-fold fuzzy set A: X→[0,1].then
i- $A \cap A^C \neq \emptyset$
ii- $A \cup A^C \neq X$

Proof:
Let $A=\{x, A_x\} \Rightarrow A^C = \{x, (1 - A_x)$ Which implies;
i. $A \cap A^C = \{x, \min(A_x, 1 - A_x)\} \neq \emptyset$
ii. $A \cup A^C = \{x, \max(A_x, 1 - A_x)\} \neq X$
We illustrate propositions 2.17 and 3.1 in example 3.3.

3.3 Example

Let A, B and C be two-fold fuzzy sets in a universe of discourse X where

X = (a, b, c} define A, B and C as follows.

A = {(a, 0.9, 0.1),(b, 0.8, 0.2),(c, 0.7, 0.3)},

B = {(a, 0.8, 0.2), (b, 0.7, 0.3),(c, 0.6, 0.4)}

C = {(a, 0.7, 0.3), (b, 0.6, 0.4), (c, 0.5, 0.5)} let x = a, b, c for simplicity. Then;

1. $A^C = \{x, (1 - \mu_A(x)), (1 - v_A(x)): x = a, b, c\} = \{(a, 0.1, 0.9), (b, 0.2, 0.8), (c, 0.3, 0.7)\}$

2. $B^C = \{x, (1 - \mu_B(x)), (1 - v_B(x)):\} = \{(a, 0.2, 0.8), (b, 0.3, 0.7), (c, 0.4, 0.6)\}$

3. $(A^C)^C = \{x, (1 - \mu_{(A^C}(x)), (1 - v_{(A^C}(x))\} = \{(a, 0.9, 0.1), (b, 0.8, 0.2), (c,0.7, 0.3)\}= A$

4. $AUA = \{x, \max(\mu_A(x)), \max(v_A(x))\} = A$

5. $A \cap A = \{x, \min(\mu_A(x)), \min(v_A(x))\} = A$

6. $AUA^C = \{x, \max(\mu_A(x), 1 - \mu_{A^C}(x)), \max(v_A(x), 1 - v_{A^C}(x))\}$

$= \{(a, 0.9, 0.9), (b, 0.8, 0.8), (c, 0.7,0.7)) \neq X$ or A

7. $A \cap A^C = \{x, \min(\mu_A(x), 1 - \mu_{A^C}(x)), \min(v_A(x), 1 - v_{A^C}(x))$

= {(a, 0.1, 0.1), (b,0.2, 0.2), (c, 0.3, 0.3)} ≠ ∅ or A^C.
8. AUB = {x, max(μ$_A$(x), μ$_B$(x)) max(v_A(x), v_B(x))} = {(a, 0.9,0.2), (b, 0.8, 0.3), (c, 0.7,0.4)}
= BUA
9. A∩B = {x, min(μ$_A$(x), μ$_B$(x)) min(v_A(x), v_B(x))} = {(a, 0.8, 0.1), (b, 0.7, 0.2),(c, 0.6, 0.3)}
= B∩A
10. AUC = {x, max(μ$_A$(x), μ$_C$(x)) max(v_A(x), v_C(x))} = {(a,0.9, 0.3), (b, 0.8, 0.4), (c, 0.7, 0.5)}
= CUA
11. A∩C = {x, min(μ$_A$(x), μ$_C$(x)) min(v_A(x), v_C(x))} = {(a, 0.7, 0.1), (b, 0.6,0.2), (c, 0.5, 0.3)}
= C∩A
12. BUC = {x, max(μ$_B$(x), μ$_C$(x)) max(v_B(x), v_C(x))})}= {(a, 0.8, 0.3), (b, 0.7,0.4), (c, 0.6, 0.5)}
= CUB
13. B∩C = {x, min(μ$_B$(x), μ$_C$(x)) min(v_B(x), v_C(x))} } = {(a, 0.7, 0.2),(b, 0.6, 0.3), (c, 0.5, 0.5)}
= C∩B
14. (AUB)^C = {x, (1 - (μ$_{AUB}$(x)), (1 - (v_{AUB}(x))}= {(a, 0.1, 0.8), (b, 0.2, 0.7),(c, 0.3, 0.6)]
15. $A^C \cap B^C$ = { x, min(μ$_{A^C}$(x), μ$_{B^C}$(x)), min(v_{A^C}(x), v_{B^C}(x))}
= {(a, 0.1, 0.8),(b, 0.2,0.7), (c, 0.3, 0.6)}
(AUB)^C = $A^C \cap B^C$
16. A∩(BUC) = {x, min(μ$_A$(x) ,(μ$_{BUC}$(x))], min($v_A(x)$, v_{BUC}(x))}
= {(a, 0.8, 0.1), (b, 0.7, 0.2), (c, 0.6, 0.3)}
17. (A∩B) U(B∩C) = {x, max(μ$_{A∩B}$ (x), μ$_{A∩C}$(x)),max($v_{A∩B}$(x), $v_{A∩C}$(x))}
= {(a, 0.8, 0.1), (b, 0.7, 0.2), (c, 0.6, 0.3)} = A∩(BUC)
18. A∩(AUB) = {x, min(μ$_A$(x) ,(μ$_{AUB}$(x)), min($v_A(x)$, v_{AUB}(x))}

= {(a, 0.9, 0.1), (b, 0.8, 0.2), (c, 0.7, 0.3)} = A

3.4 Discussion

The Union operator so far defined is monotonic, commutative, associative, continues and idempotent by; 2.23, 3.1(2, 3, and 4), and 3.3, Thus is a fuzzy union. Similarly, the intersection operator is also fuzzy from 2.23, 3.1 (2, 3 and 4) and 3.2. Furthermore, the complement of two-fold fuzzy set is as well monotonic, involutivity by 3.1(1) and continuous and therefore is a fuzzy complement.
Having satisfied the basic operations of fuzzy union, intersection, complementation, we have that:
the two-fold fuzzy set satisfied De-Morgan's law, Absorption, and Distributivity, based on 3.1 (9, 5 and 6, and 8 respectively). Like the parent set, the two-fold fuzzy set does not satisfy the law of excluded middle and law of contradiction as shown in 3.2. Therefore, the set can be used in a similar way to solve any imprecision and vague problems with better expected outcome as dual membership is explored and leveraged on.

4.4 Summary

The work examined the conformity of the two-fold fuzzy sets with the existing Boolean algebra in the theory of fuzzy sets. The basic operations of the traditional fuzzy set were investigated against the two-fold fuzzy sets. The axioms of fuzzy union, fuzzy intersection and fuzzy complement were investigated and extended to two-fold fuzzy set. The set having satisfied the axioms was proved to be commutative, associative, distributive, involutivity, absorptive point-wise, and in concord with De Morgan's law. However, it fails the laws of excluded middle and contradiction.

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