



Concepts for Elementary School Mathematics

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Abstract:

Continuing studies show that fifty percent, or less, of school age children are working at grade level in mathematics. This paper presents some ideas and concepts that may help school age children gain mastery of early mathematical principles.

Keywords:

Elementary school mathematics, integer operations, even and odd integers, closure properties, mathematical proofs.



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1. Introduction

For countless years, students in elementary school, middle school and high school have taken standardized tests over subject matter, notably reading and mathematics. For just as long, the standardized tests have shown that fifty percent of students, or fewer, are actually working at grade level in reading and mathematics.

This paper presents some ideas and concepts that we believe should be incorporated into the mathematics curriculum. These concepts deal with addition and multiplication of integers. Typically, children study their multiplication tables. However, multiplication tables go only so far. These concepts are designed to help students understand the results of addition and multiplication of integers and provide early indications of correct results of addition and multiplication of integers.

Though students may master the addition and multiplication tables, the tables are only ten by ten or twelve by twelve, somewhat limited. The tables do not include something as simple as eight plus fourteen or eight times fourteen. The concepts presented here can give an early indication of whether a result will be correct or not.

2. Background

The concepts presented here are remarkably simple. But before presenting the concepts, we must present our definition:

Definition: An integer a is an even integer provided that there exists an integer n such that $a = 2n$. An integer a is an odd integer provided there exists an integer n such that $a = 2n + 1$.

It is also necessary that we be familiar with the closure properties of integers:

Closure: Let Z stand for the set of integers. Three of the basic properties of the integers are that the set Z is closed under addition, the set Z is closed under multiplication and the set Z is closed under subtraction.

This means that:

- a. If x and y are integers, then $x + y$ is an integer.
- b. If x and y are integers, then $x * y$ is an integer
- c. If x and y are integers, then $x - y$ is an integer.

Given our definition and understanding of the closure properties of integers, we can proceed with concepts.

3. Concepts

The concepts and their proofs are presented here.

Concept 1: The sum of an even integer and an even integer is an even integer.

Proof: Let $a = 2n$ and $b = 2m$. Then $a + b = 2n + 2m = 4nm = 2(2nm)$ which can be written as $2k$ by the closure properties. By definition, this is an even integer.

Concept 2: The sum of an even integer and an odd integer is an odd integer.

Proof: Let $a = 2n$ and $b = 2m + 1$. Then $a + b = 2n + 2m + 1 = 2(n + m) + 1$, which can be written as $2k + 1$ by the closure properties. By definition, this is an odd integer.

Concept 3: The sum of an odd integer and an odd integer is an even integer.

Proof: let $a = 2n + 1$ and $b = 2m + 1$. Then $a + b = 2n + 2m + 2 = 2(n + m + 1)$ which can be written as $2k$ by the closure properties. By definition, this is an even integer.

Concepts 4: The product of an even integer and an even integer is an even integer.

Proof: Let $a = 2n$ and $b = 2m$. Then $a * b = 2n * 2m = 2(n * m)$ which can be written as $2k$ by the closure properties. By definition, this is an even integer.

Concept 5: The product of an even integer and an odd integer is an even integer.

Proof: Let $a = 2n$ and $b = 2m + 1$. Then $a * b = 2n * 2m + 2n = 2(n * m + 2)$ which can be written as $2k$ by the closure properties. By definition, this is an even integer.

Concept 6: The product of an odd integer and an odd integer is an odd integer.

Proof: Let $a = 2n + 1$ and $b = 2m + 1$. Then $a * b = (2n + 1) * (2m + 1) = 4nm + 2n + 2m + 1 = 2(2nm + n + m) + 1$ which can be written as $2k + 1$ by the closure properties. By definition, this is an odd integer.

Concept 7: An even integer minus an even integer is an even integer.

Proof: Let $a = 2n$ and $b = 2m$. Then $a - b = 2n - 2m = 2(n - m)$ which can be written as $2k$ by the closure properties. By definition, this is an even integer.

Concept 8: An odd integer minus an even integer is an odd integer.

Proof: Let $a = 2n + 1$ and let $b = 2m$. Then $a - b = 2n + 1 - 2m = 2(n - m) + 1$ which can be written as $2k + 1$ by the closure properties. By definition, this is an odd integer.

Concept 9: An odd integer minus an odd integer is an even integer.

Proof: Let $a = 2n + 1$ and $b = 2m + 1$. Then $a - b = 2n + 1 - 2m - 1$ which can be written as $2k$ by the closure properties. By definition, this is an even integer.

Concept 10: An even integer minus an odd integer is an odd integer.

Proof: Let $a = 2n$ and $b = 2m + 1$. Then $a - b = 2n - 2m - 1$ which can be written as $2k - 1$ by the closure properties. By definition, this is an odd integer.

4. Conclusion

Though the above concepts may seem simple and obvious to most people, they can be helpful to the budding mathematician. If students know the above concepts, they can perhaps quickly determine if an addition or multiplication is going awry.

5. References

1. T. Sundstrum, "Mathematical Reasoning: Writing and Proof, Version 2.1", May 2020.